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Technical Note

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## ATLAS OF FOURIER COEFFICIENTS OF DIURNAL VARIATION OF foF2

WILLIAM B. JONES



U. S. DEPARTMENT OF COMMERCE NATIONAL BUREAU OF STANDARDS

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# NATIONAL BUREAU OF STANDARDS Echnical Mote

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**APRIL 1962** 

### ATLAS OF FOURIER COEFFICIENTS OF DIURNAL VARIATION OF 60F2

William B. Jones
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#### ABSTRACT

A series of graphical representations is given for illustrating the regular and continuous geographic variations of Fourier coefficients a; and b; obtained from the diurnal analysis of foF2 monthly medians, including their main latitudinal trend, mixed latitudinal and longitudinal variation, and the effect due to noise (random fluctuation in the original data). To illustrate the systematic changes in these variations with seasons and with solar activity, corresponding graphs are given for four seasonal months for minimum and maximum years of solar activity (1954 and 1958).

### ATLAS OF FOURIER COEFFICIENTS OF DIURNAL VARIATION OF foF2

	CONTENTS	
	-	Page
1.	INTRODUCTION	1
2.	THE BASIC INPUT DATA	3
3.	FOURTER ANALYSIS	5
4.	DESCRIPTION OF THE GRAPHICAL REPRESENTATIONS	7
	ACKNOWLEDGEMENTS	11
5•	REFERENCES	12
	GRAPHICAL REPRESENTATIONS	
	Constant Term a	14-17
	First Harmonic	18-25
	Second Harmonic	26-33
	Third Harmonic	34 <b>-4</b> 1
	Fourth Harmonic	42-49
	Fifth Harmonic	50-57
	Sixth Harmonic	58 <b>-</b> 65
	Seventh Harmonic	66-73
	Eighth Harmonic	74-81
	Ninth Harmonic	82-89
	Tenth Harmonic	90-97

Eleventh Harmonic

98-105

#### 1. INTRODUCTION

A solution has recently been given [Jones and Gallet, 1962] to the problem of representing the complex properties of ionospheric characteristics on a world-wide scale, including their diurnal variation, by numerical analysis of ionospheric data as they are measured at the stations, without prior hand operations. In the solution referred to above, the analysis is made in two main parts. (1) The diurnal analysis is performed by Fourier analysis of the 24 hourly measurements from each available station. The resulting Fourier coefficients are then corrected to local mean time (IMT). (2) The geographic analysis is made by expanding the world-wide geographic variation of each Fourier coefficient in an orthonormal series of functions of latitude and longitude, which are linear sums of surface spherical harmonics. Special filtering processes are employed in each part to obtain an optimum separation of noise (random fluctuation of the ionospheric data) from the real physical variations.

The Fourier (time series) coefficients are corrected to LMT so that their main geographic variation becomes latitudinal. In addition to the main latitudinal trend; however, there is also a less important

For a brief summary of the problem and solution, the reader can refer to Jones and Gallet [1960].

but physically significant mixed latitudinal and longitudinal variation. Superimposed on the real physical variations is an effect due to noise which is small in amplitude but which becomes the dominant part in the high order harmonics.

The purpose of this atlas is to illustrate - by means of a series of graphs - the variations mentioned above and to show their systematic changes with seasons and with solar activity. The regularity and continuity of these phenomena were essential to the solution of numerical mapping which has been given. Although more objective methods (such as numerical analyses and statistical tests for significance (have been used for studying the physical variations and the effects of noise, graphical representations of the type given here have guided the development of the mapping methods and have added insight and understanding to the complex physical properties of the ionosphere to be represented.

The ionospheric characteristic which we have used here is the monthly median of the F2-layer critical frequency (foF2), partly for the purpose of illustration but also since it is the most interesting and important from the viewpoint of practical applications of radio propagation.

Similar studies have also been made for other characteristics such as the 3000 km maximum usable frequency factor (F2-M3000). For this case the physical variations are much weaker than for foF2, but regular and continuous variations are again observed as well as systematic long-term changes due to seasons and the eleven year solar cycle.

#### 2. THE BASIC INPUT DATA

The basic input data used for the present series of graphs consist of the 24 hourly medians of foF2 from all available ionospheric stations for the four seasonal months, March, June, September, and December, for each of the last minimum and maximum years of solar activity (1954 and 1958). Table 1 gives the total number of stations available at the time this study was made and the Zurich sunspot numbers for each month. The data were punched on cards as they had been tabulated in ionospheric booklets (or in some cases on microfilm) without prior smoothing. In a few cases it was necessary to reject data from certain stations if, for example, so many consecutive hourly values were missing that the diurnal curve could not be defined, or if the data were grossly inconsistent with those from closely neighboring stations. For the IGY year (1958) there were about twice as many stations as for 1954, and so the world-wide coverage was considerably better, particularly in the southern hemisphere.

TABLE 1

	1954				1958			
	Mar.	Jun.	Sep.	Dec.	Mar.	Jun.	Sep.	Dec.
Number of Available Stations	66	68	63	64	118	118	113	118

Zurich Sunspot Numbers								
A. Monthly Mean	10.9	0.2	1.5	7.6	190.7	171.5	201.2	187.6
B. 12 Month Running Average	4.2	4.2	7.8	12.0	201.3	186.8	183.8	180.5

Average Standard Deviation o <sub>l</sub> (Mc) for High Order (noise) Harmonics								
For all Available Stations	0.049	0.043	0.055	0.055	0.072	0.069	0.084	0.078
For Selected * Latitude Bands:								
60° < λ ≦ 90°	0.030 (7)	0.016			0.092 (28)		0.075 (28)	0.087 (27)
30° < λ ≦ 60°	0.032 (26)	0.030	0.040 (23)	0.043 (23)		0.035 (39)	0.058 (42)	0.064
-30° < λ ≤ 30°	0.069	0.061	0.081	0.075 (24)		0.084	0.10 (24)	0.087 (34)
-60° < λ ≦ -30°	0.043	0.040	0.035 (8)	0.040	0.066	0.10 (10)	0.090	0.072 (12)
-90° ≤ λ ≤ -60°	0.032	0.021	0.038	0.035	0.072 (12)	0.12 ( 9)	0.12	0.049

<sup>\* (</sup>Numbers in parentheses indicate numbers of stations contained within the particular latitude band.)

#### FOURIER ANALYSIS

For each station (Table 1) a Fourier analysis is made of the 24 hourly medians of foF2, thus decomposing the diurnal variation into eleven harmonics 1

$$a_{j} \cos jt + b_{j} \sin jt = c_{j} \cos (jt - \psi_{j})$$
 (1)  
 $j = 1, 2, ..., 11$ 

and a constant term a which is the arithmetic mean of the data. The variable t is used to denote the local mean hour angle which is related to LMT (in hours) by

$$t = 15^{\circ} (IMT) - 180^{\circ}.$$
 (2)

For example, at noon LMT, t = 0°.

The coefficients  $a_j$  and  $b_j$  in (1) are related to the amplitude  $c_j$  and phase  $\psi_j$  by

$$a_{j} = c_{j} \cos \psi_{j}$$

$$b_{j} = c_{j} \sin \psi_{j}$$

$$c_{j} = \sqrt{a_{j}^{2} + b_{j}^{2}}$$

$$\psi_{j} = \arctan \frac{b_{j}}{a_{j}}.$$
(3)

The essential formulas for computing the  $a_j$ ,  $b_j$ ,  $c_j$ , and  $\psi_j$  are given by Jones and Gallet [1962]. The final term  $a_{12}$  cos 12 t has been omitted since it does not form a complete harmonic and also since it is produced mainly by noise.

Although the graphs in this atlas are restricted to the coefficients  $a_j$  and  $b_j$  plotted against geographic latitude, with the help of (3) one can visualize approximately the corresponding variations of  $c_j$  and  $\psi_j$ . For example, the quadrant of the phase  $\psi_j$  can easily be deduced from the arithmetic signs of  $a_j$  and  $b_j$  (see Table 2).

TABLE 2

Location of the Phase  $\psi$ 

(a <sub>j</sub> , b <sub>j</sub> )	Quadrant of the phase $\psi_{\mathbf{j}}$
(-,-)	-180° < ψ <sub>j</sub> < - 90°
(+ , -)	-90° <ψ <sub>j</sub> < 0°
(+ , + )	0° <ψ <sub>j</sub> < 90°
(-,+)	90° < ψ <sub>j</sub> < 180°

As an illustration we see from the graphs of  $a_1$  and  $b_1$  for September 1958 (pages 18 and 19) that both coefficients are mostly positive, thus showing that the phase lies in the range  $0^{\circ} < \psi_1 < 90^{\circ}$  (from which we conclude that the maximum value of the first harmonic occurs shortly after noon as one would expect). On the other hand from the graphs of  $a_2$  and  $b_2$  for the same month (pages 26 and 27) we see that  $a_2$  is mostly positive and  $b_2$  mostly negative, so that the phase for the second harmonic falls in the range  $-90^{\circ} < \psi_2 < 0^{\circ}$ .

#### 4. DESCRIPTION OF THE GRAPHICAL REPRESENTATIONS

#### World-Wide Geographic Variations

The type of graph used here consists of the Fourier coefficients a; (and b;) from all available stations for a given month plotted against the station latitude  $(\lambda)$ , so that the main latitudinal trend is emphasized. Such graphs are given for all months shown in Table 1, and for all coefficients  $a_0$ ,  $a_i$ , and  $b_i$ , j = 1, 2, ..., 11 (pages 14)to 105). Special plotting symbols have been used to signify the approximate station longitudes  $(\theta)$ , so that in a sense the graphs are three dimensional. As a result, it can be seen that the coefficients are not randomly dispersed about the main latitudinal trend, but -- for some harmonics -- they are systematically arranged according to longitude. This effect is illustrated clearly, for example, in Figure 1, which is an enlargement of the graph of a for June 1958. In this graph the points in the northern hemisphere (20°N to 80°N) appear to fall into one of two distinct groups. Almost without exception the points in the lower group are from stations with longitudes between 225°E - 345°E, and the points in the upper group are from stations outside this longitude band. It can be seen from the graph of a for December 1958 that the reverse effect occurs in the southern hemisphere. The same effect occurs in March and September 1958 (to a lesser degree) and also in 1954.

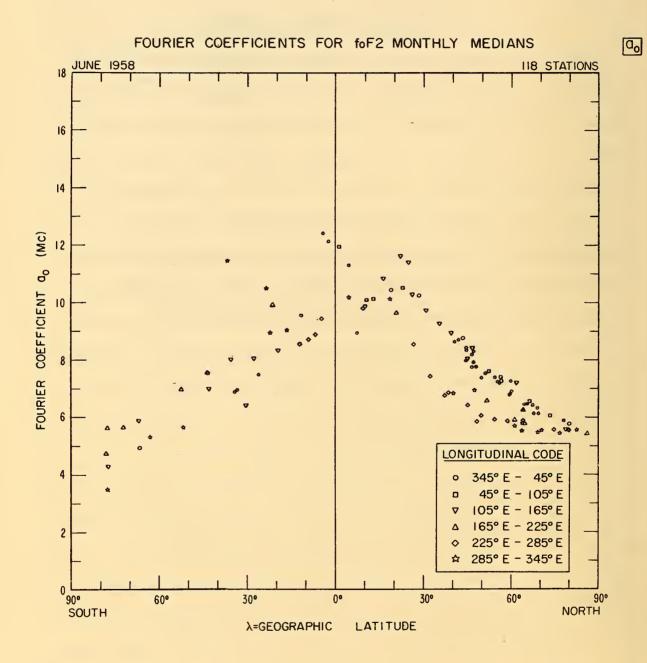


Figure 1

#### Changes with Seasons and Solar Activity

The series of graphs given here have been grouped by harmonics, by months, and by years. In order to illustrate the strong seasonal changes we have grouped the two equinox months (March and September) and the two opposing solstice months (June and December). These seasonal changes are illustrated, for example, in the graphs of all and blue by the equatorial symmetry for March and September and by the similarity and consistency of the graphs for these two months, and by the asymmetry and reverse effects for June and December. The effect of change in solar activity is seen mainly in the large differences in amplitudes of corresponding graphs for 1954 and 1958.

#### Effect of Noise

The effect of noise (random fluctuation) in the original data on the Fourier coefficients  $a_j$  and  $b_j$  is well known. It can be shown, in fact, [Chapman and Bartels, 1940] that the Fourier coefficients  $a_j$  and  $b_j$  obtained from the analysis of random numbers  $y_1, y_2, \dots, y_{24}$  -- where the values  $y_i$  are independently, normally distributed with mean zero and standard deviation  $\sigma$  - have independent normal distributions with mean zero and standard deviation

$$\sigma_1 = \frac{\sigma}{\sqrt{12}} = 0.289 \ \sigma.$$
 (4)

The effect due to noise is illustrated by the present series of graphs. Although the noise affects all harmonics by approximately the same amount, its effect is felt much more strongly in the high order harmonics where the amplitude of the physical variation is below the noise level. For foF2 medians the harmonics above order 8 are produced mainly by noise, whereas the lower harmonics (with the possible exception of 7 and 8) represent mostly real physical variation [Jones and Gallet, 1962]. Thus, as would be expected (from the discussion above), the strong geographic variations seen in the graphs of the low order coefficients a, and b, gradually give way to the random effect due to noise in the high frequencies. In fact, it can be seen from the graphs of a and b (j = 9, 10, 11) that the points are approximately symmetrically scattered around zero in a more or less random manner. The spread of the points about zero, however, appears to vary with latitude, suggesting a variation of  $\sigma_1$  (and hence of the noise  $\sigma$ ) with geographic position.

The average standard deviation  $\sigma_1$  has been computed from (4) for each of the months under consideration, the noise  $\sigma$  in the original foF2 medians being obtained from (41) in [Jones and Gallet, 1962]. Table 1 contains the values of  $\sigma_1$  for all available stations taken together as well as for selected latitudinal bands, so that the variation in latitude can be seen. These values give not only the average standard deviation of the high order (noise) coefficients but also an estimation of the effect of noise in the low order harmonics.

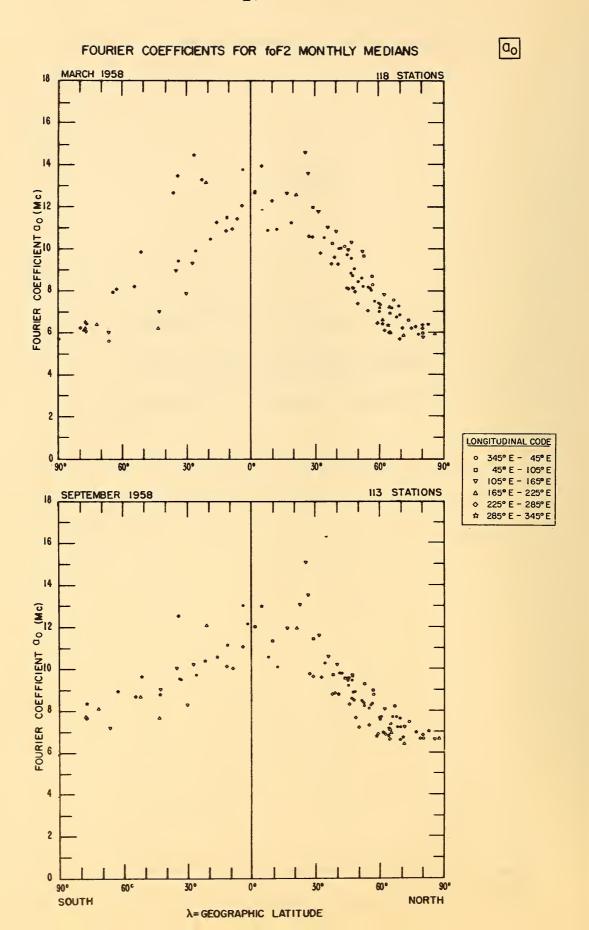
#### ACKNOWLEDGEMENTS

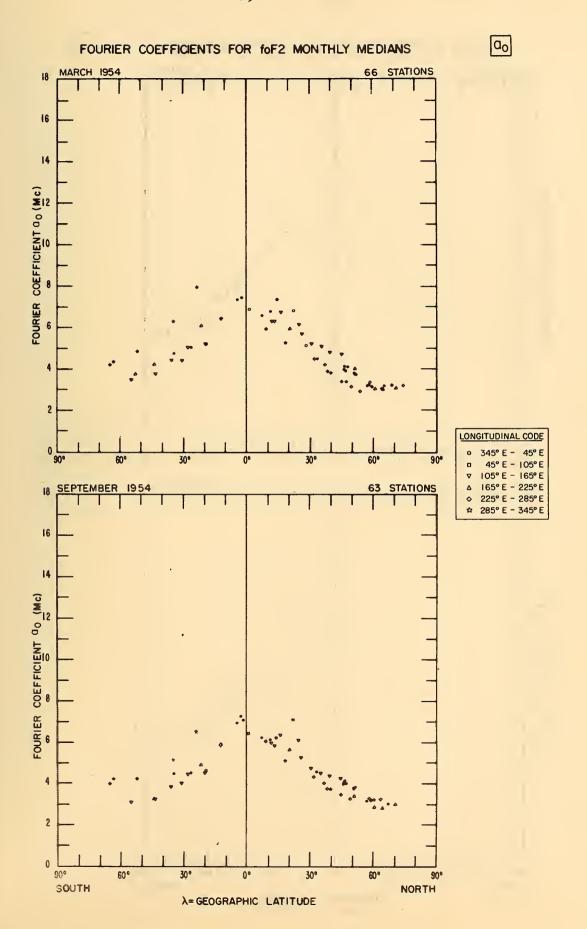
The author gratefully acknowledges the assistance received from a number of persons. The computer programming was performed by Mrs. Virginia L. Rios. The many graphs given in the present atlas were rendered possible by the careful work of Mrs. J. Kaye Myers. She was responsible for the preparation of the ionospheric data on punch cards, the application of computer programs to obtain the Fourier coefficients, and the plotting and preparation of the graphs in the present form. Able assistance in assembling the material and in typing the manuscript was given by Mrs. Anna von Kreisler. Other persons contributing directly to this publication include the operators of the Central Computing Facility of the N.B.S. Boulder Laboratories.

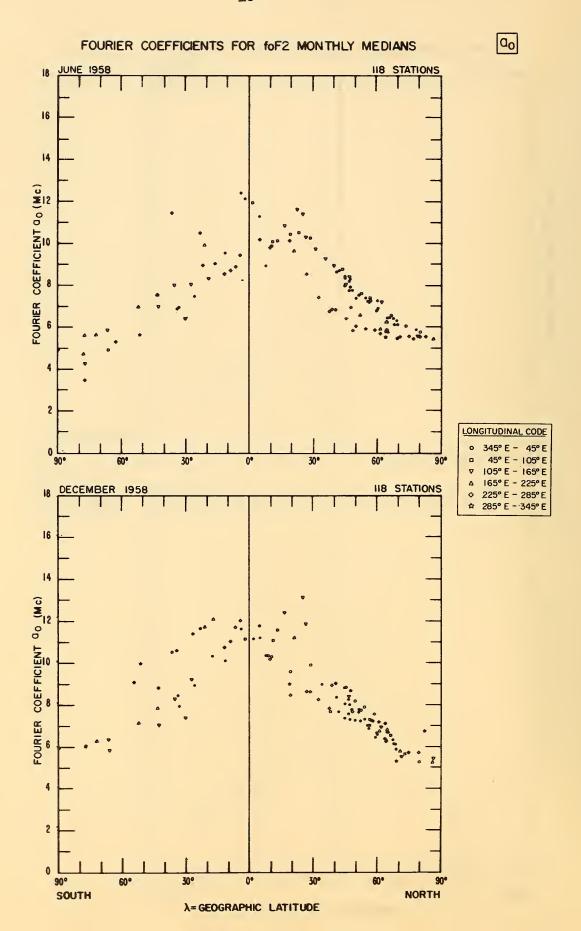
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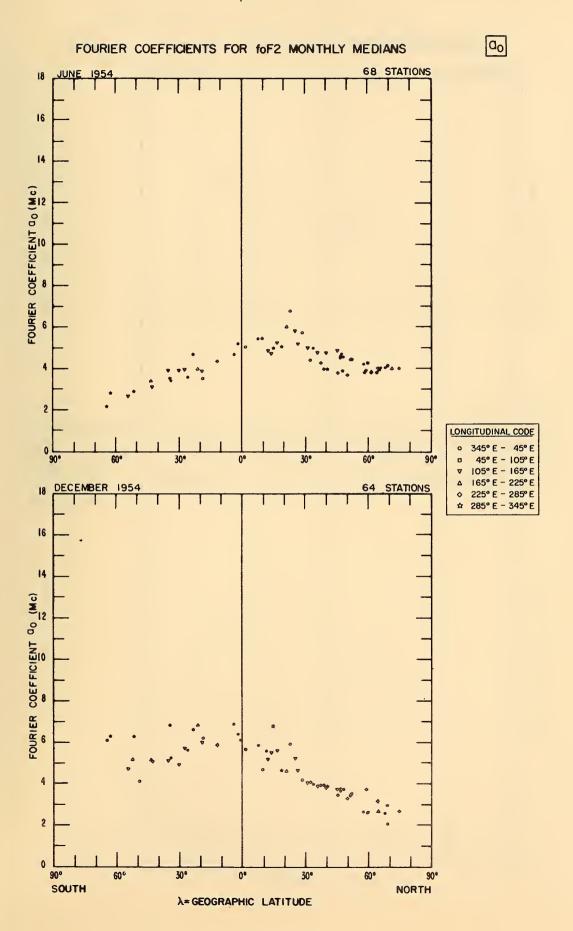
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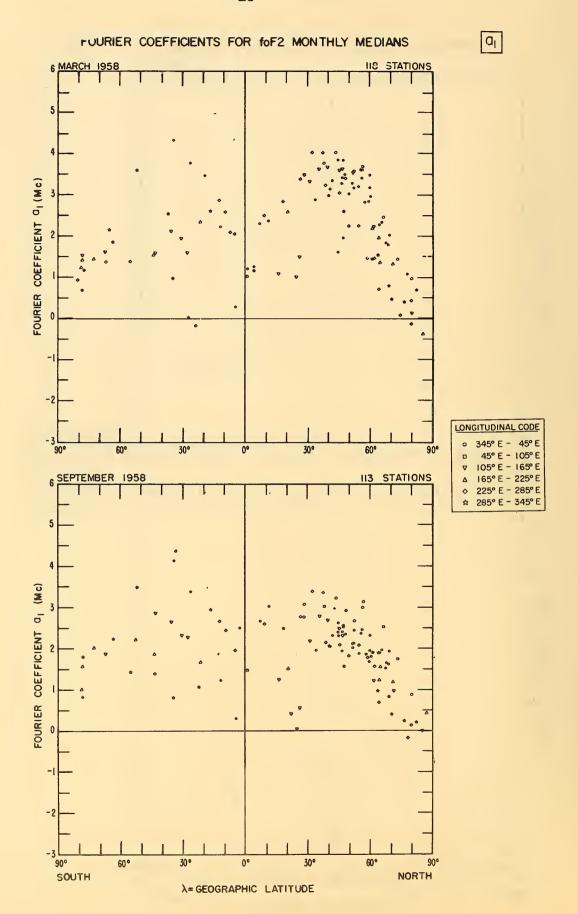
SERIES OF GRAPHICAL REPRESENTATIONS OF FOURIER COEFFICIENTS a, AND b, FROM THE DIURNAL ANALYSIS OF foF2 MEDIANS

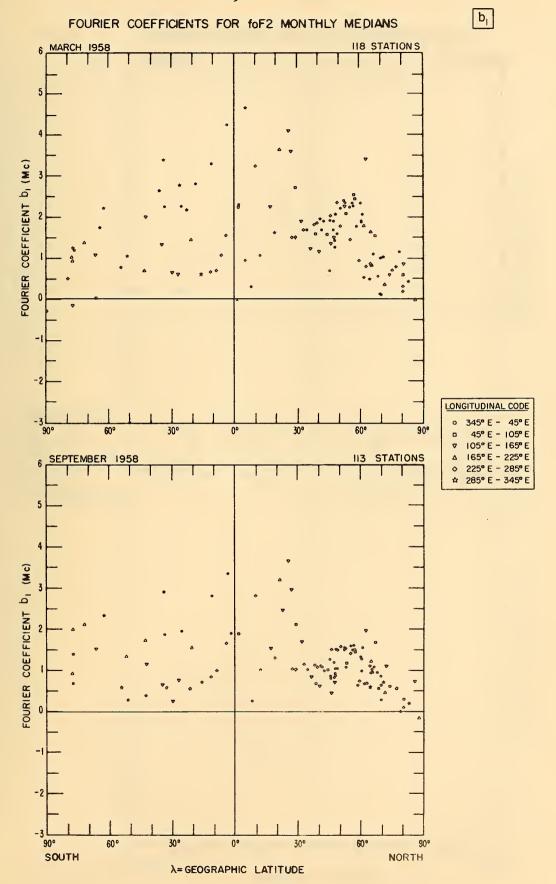


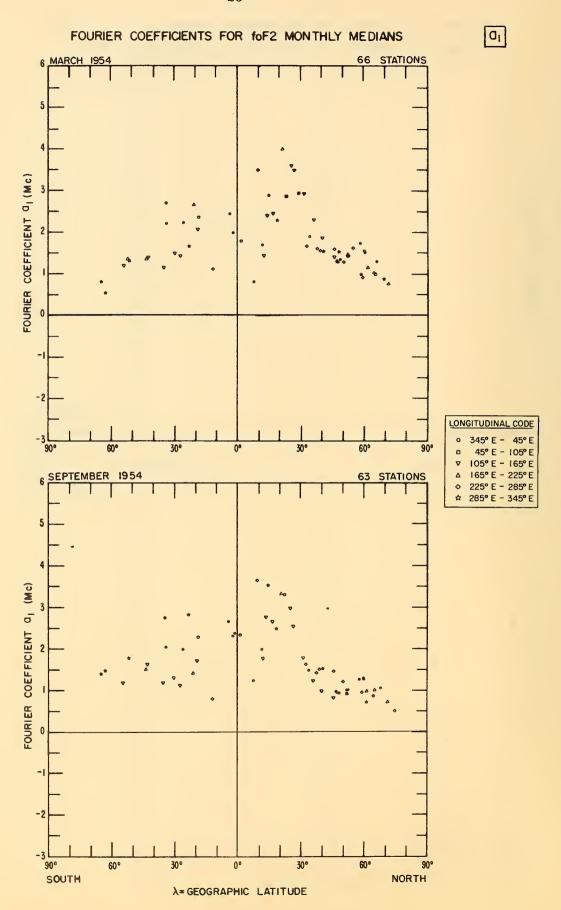


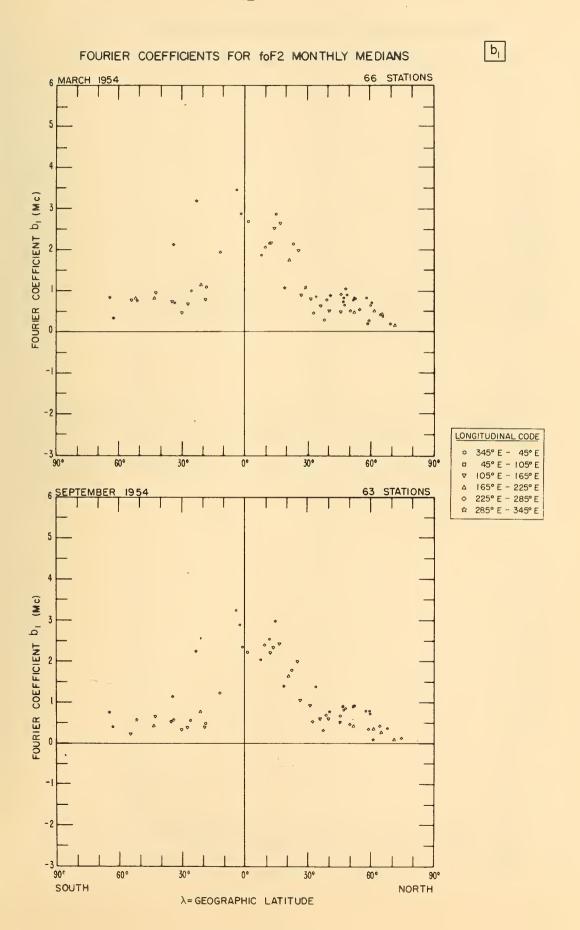


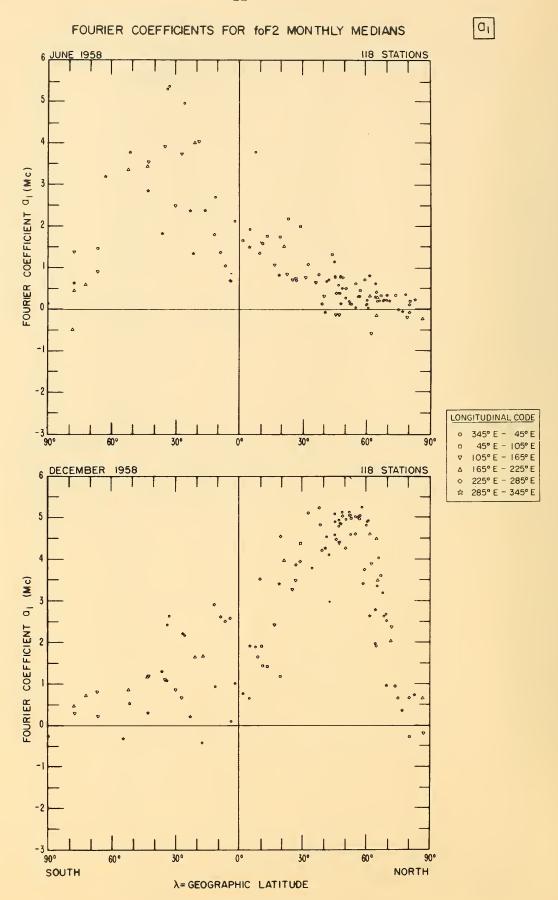


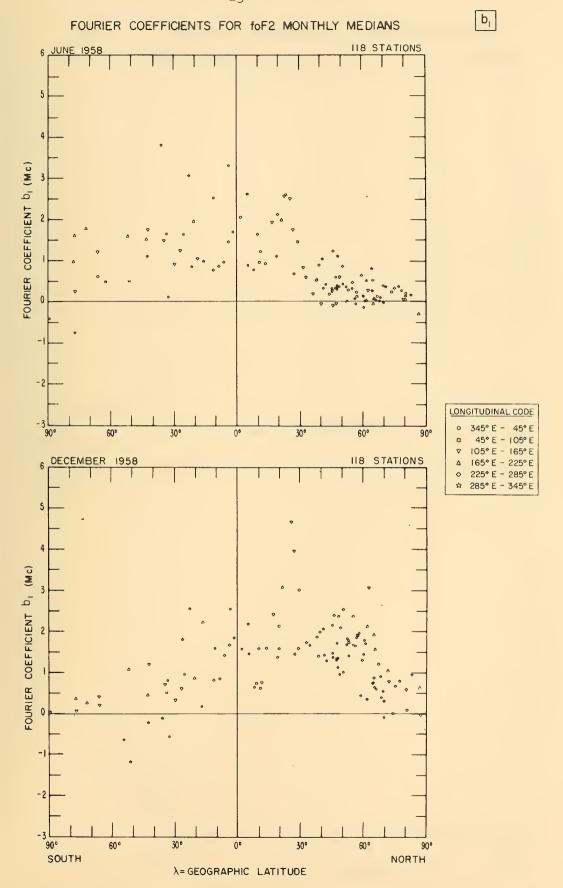


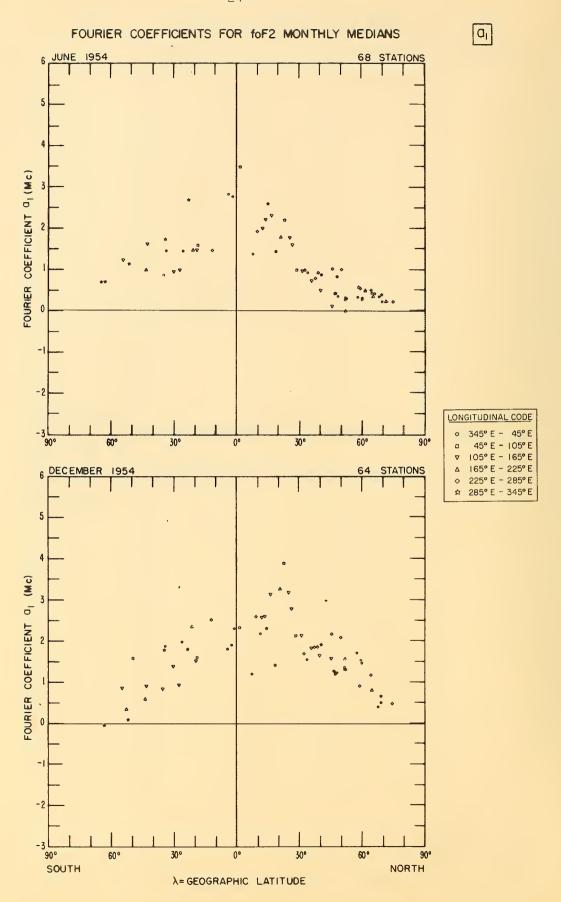


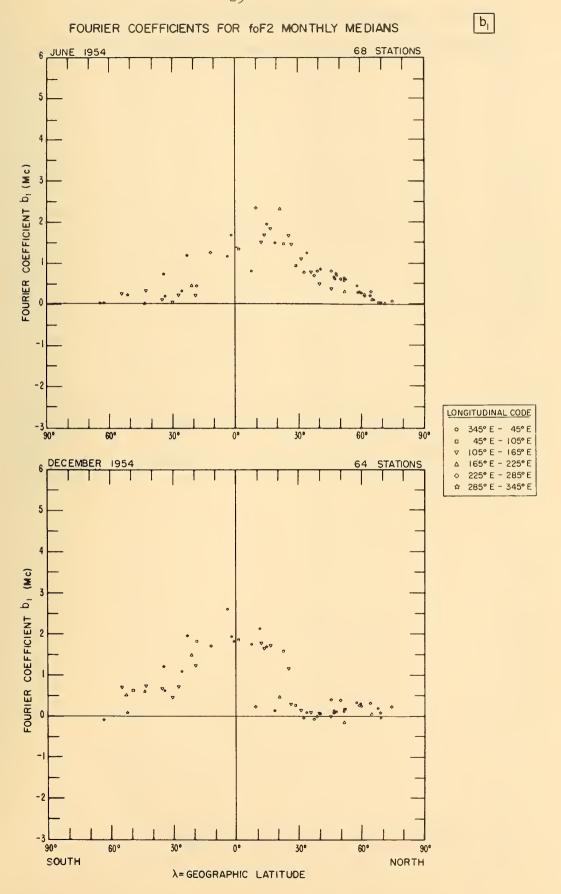


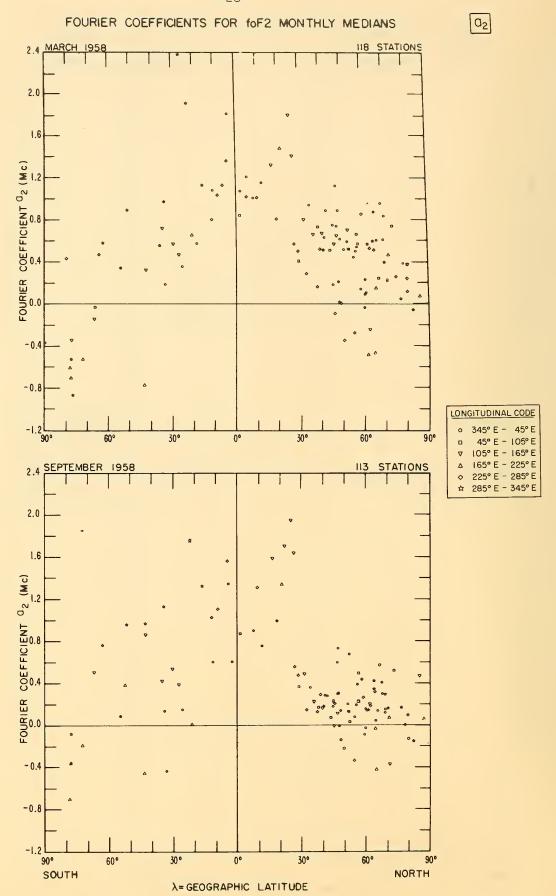


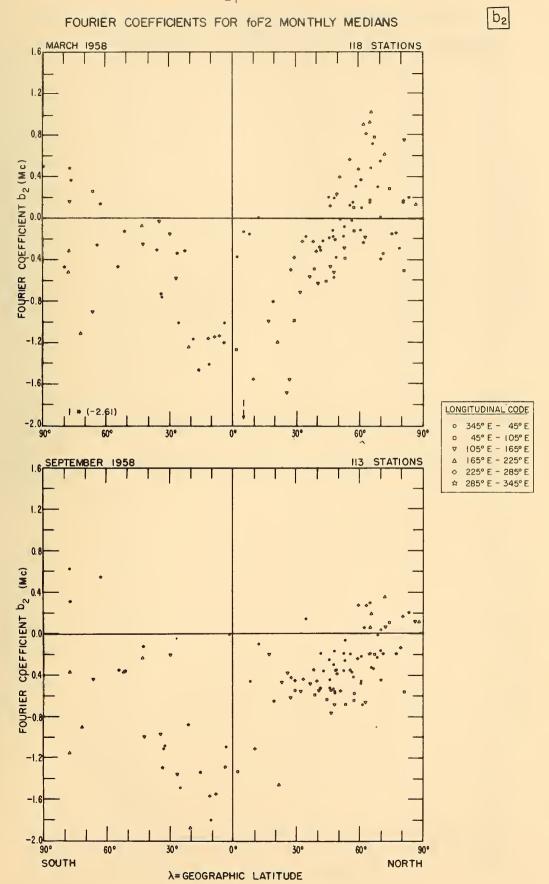


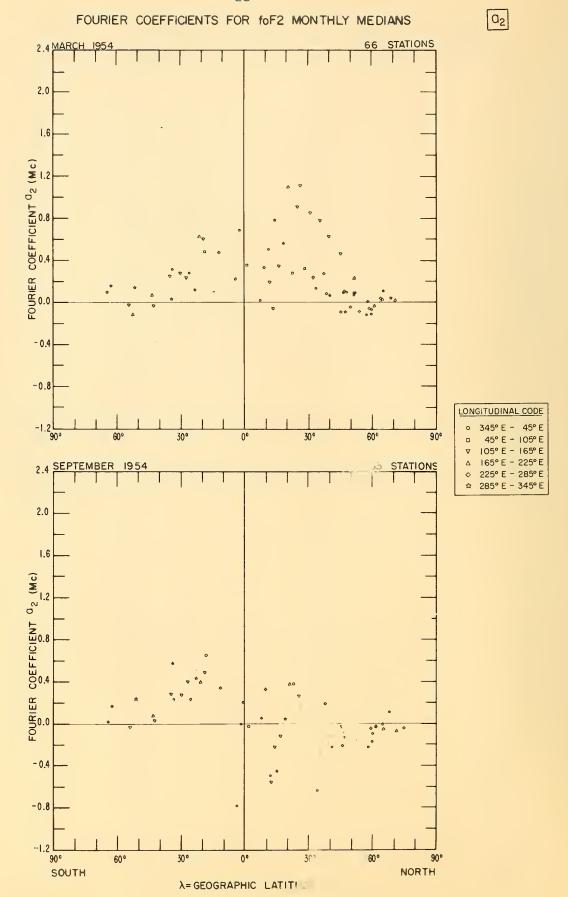


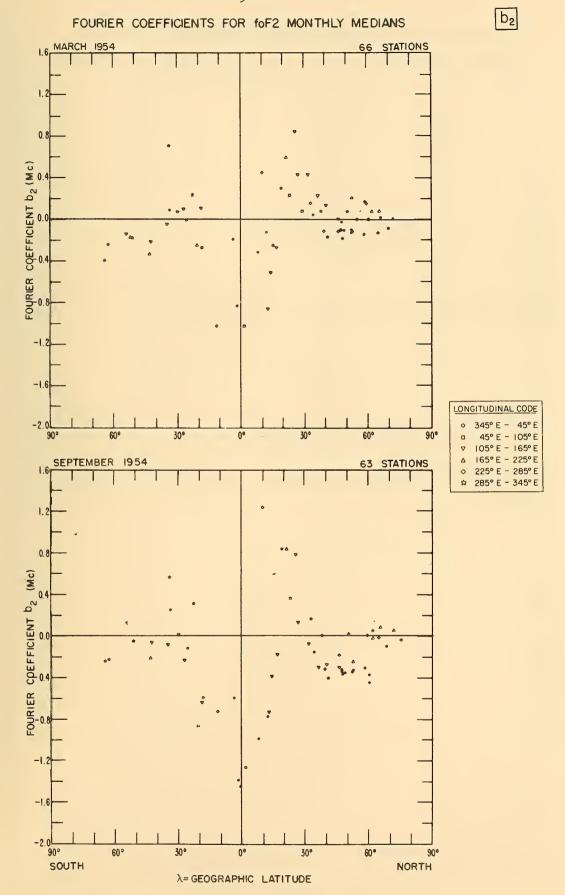


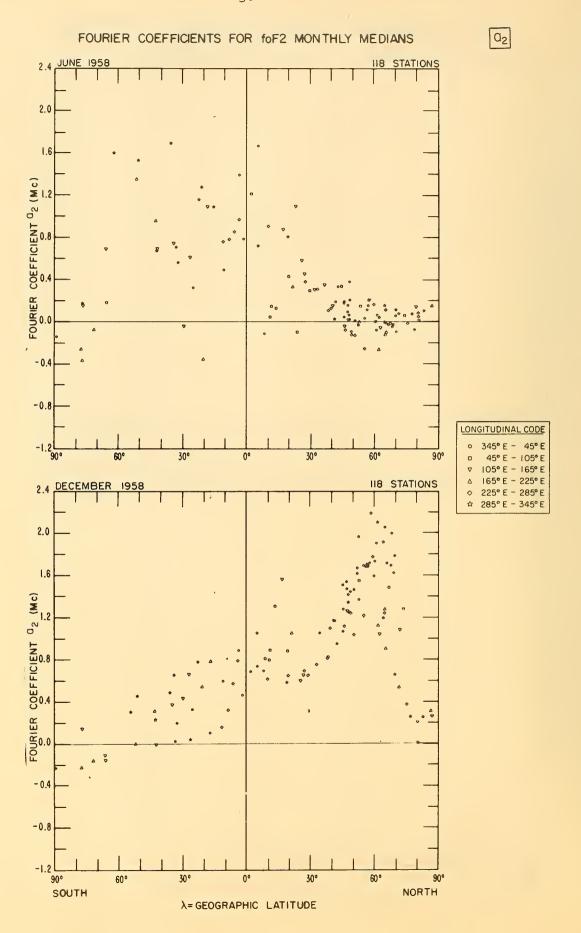


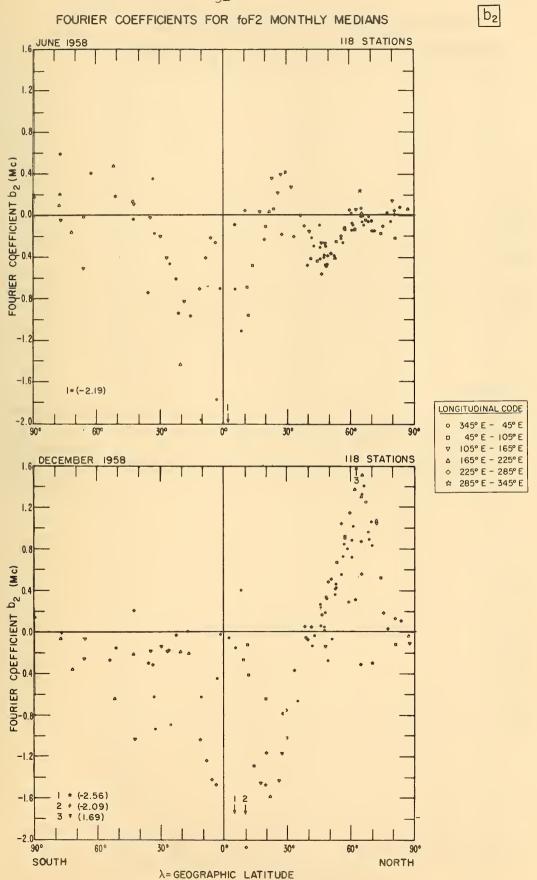


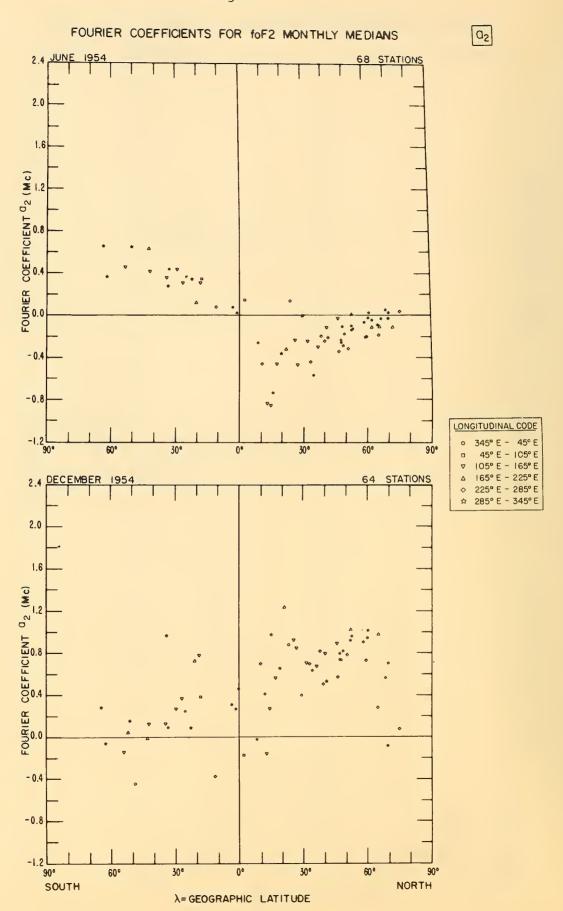


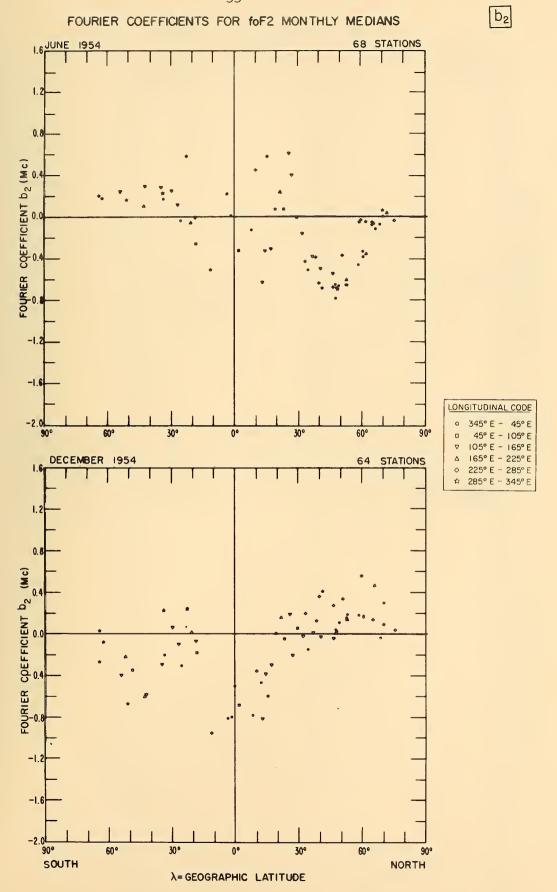


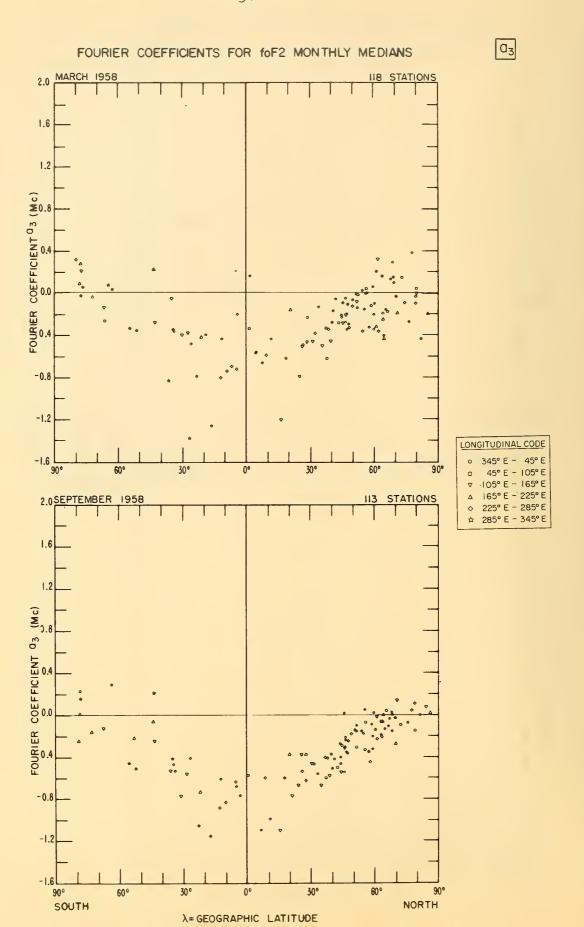


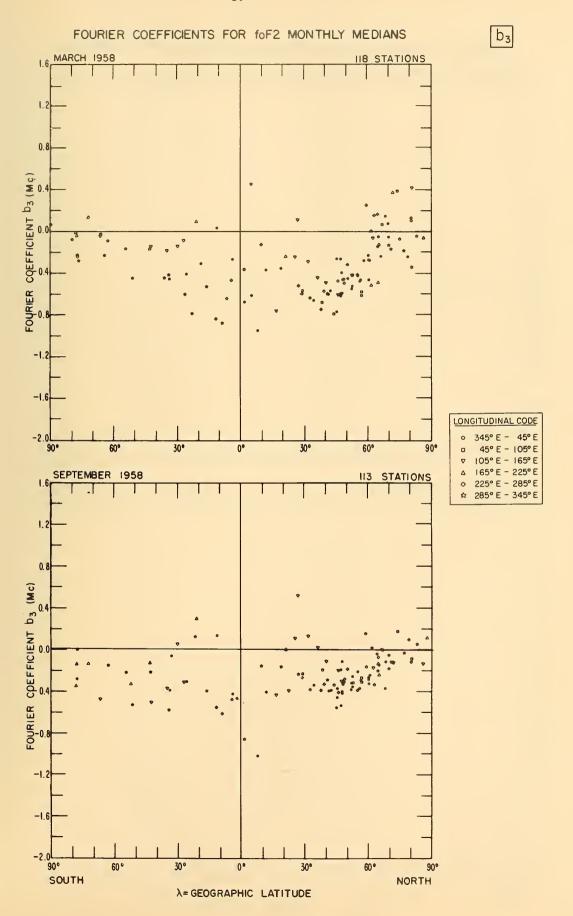


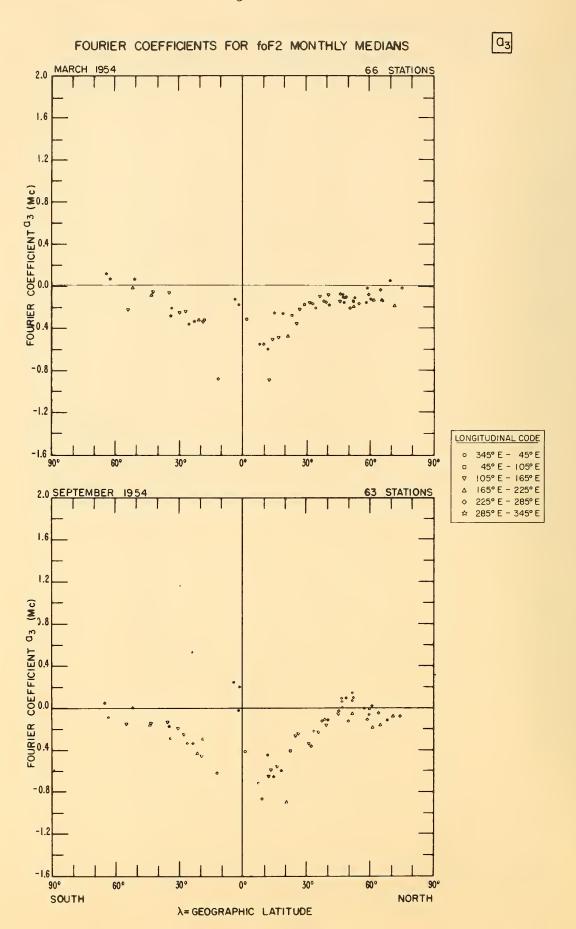


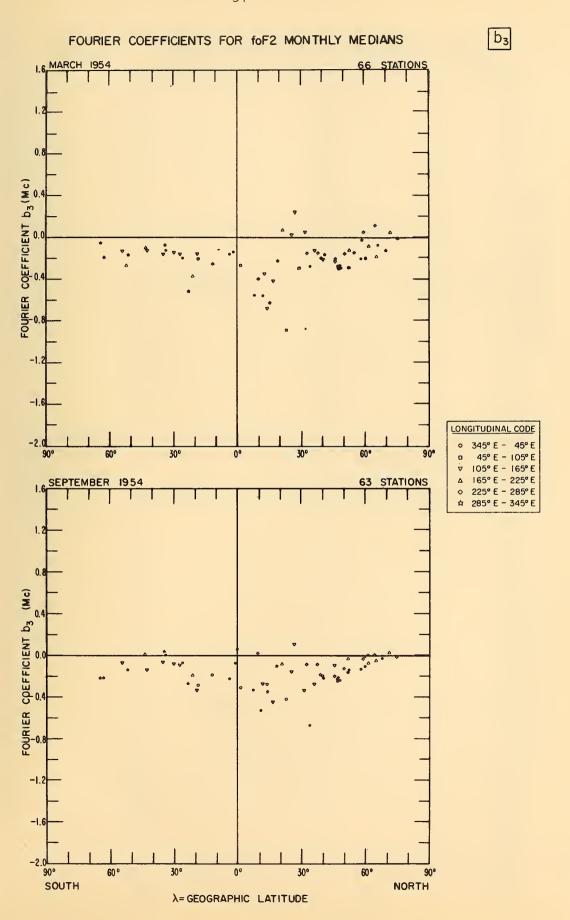


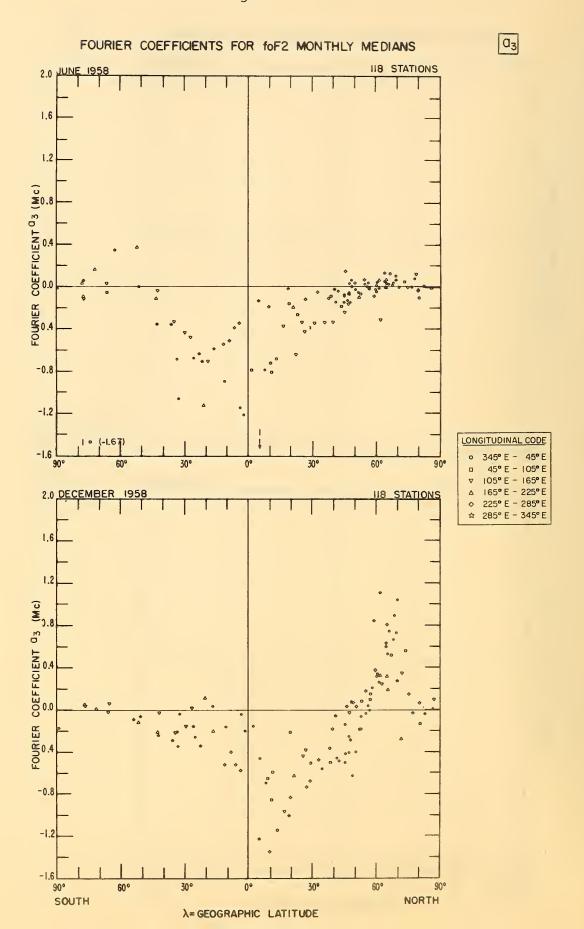


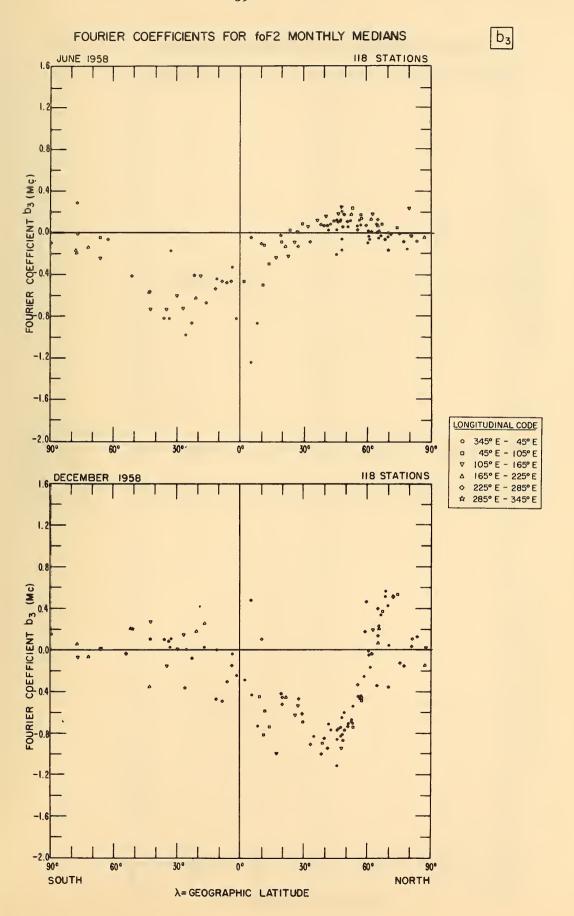


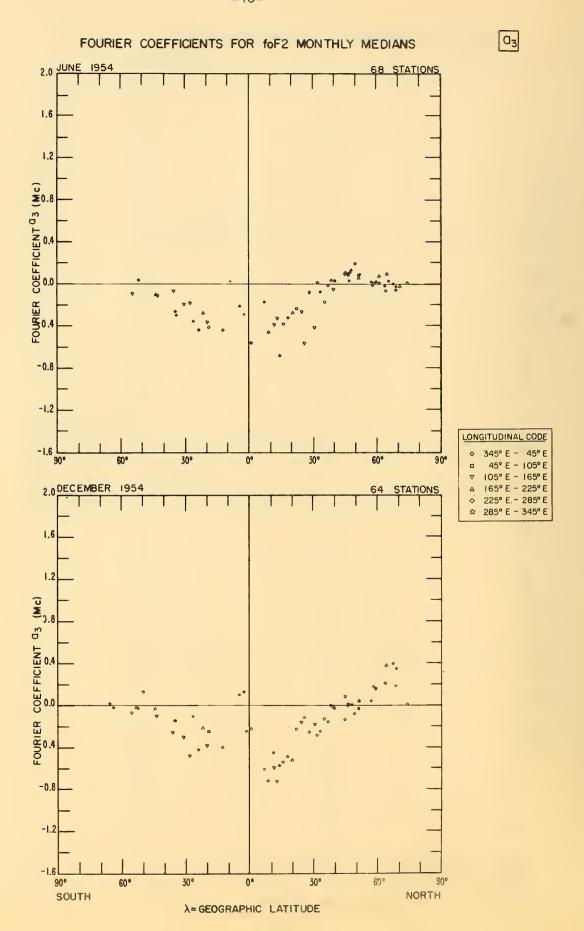


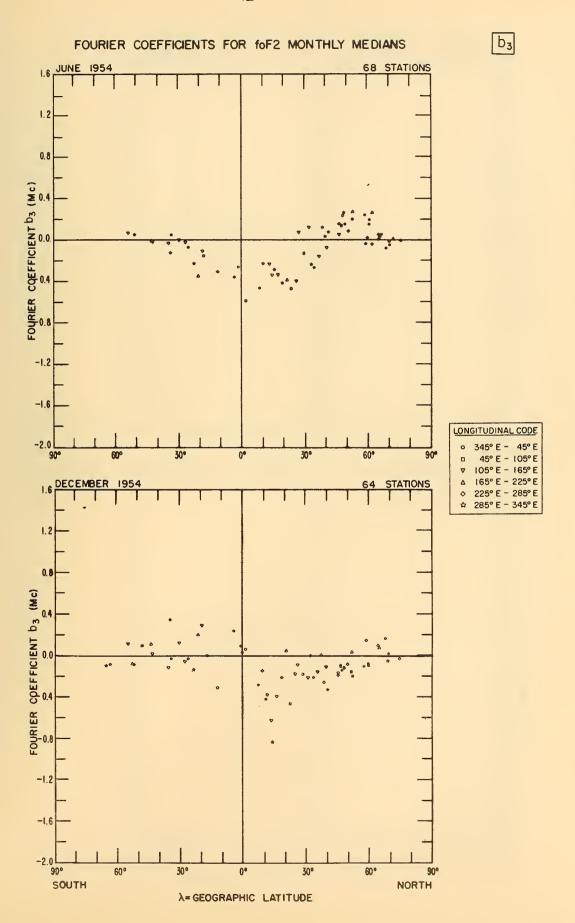


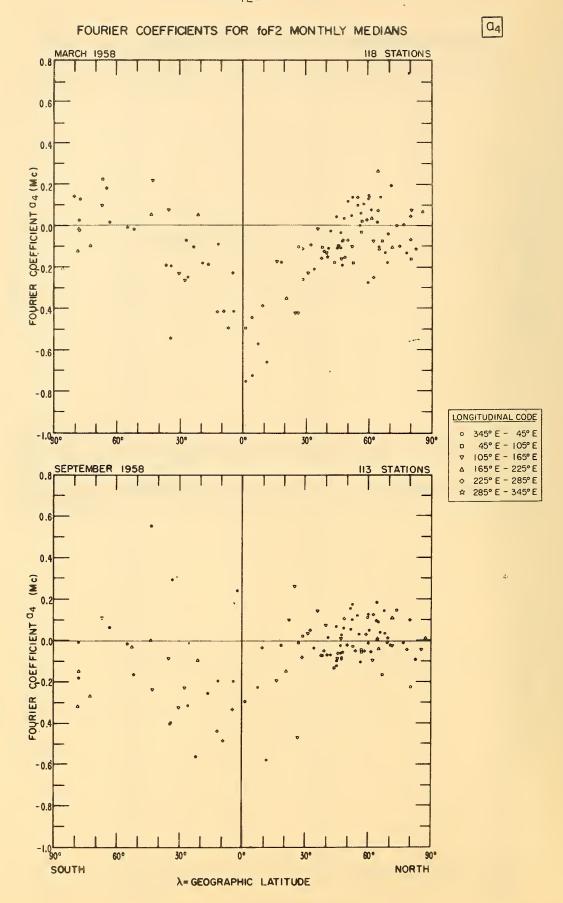


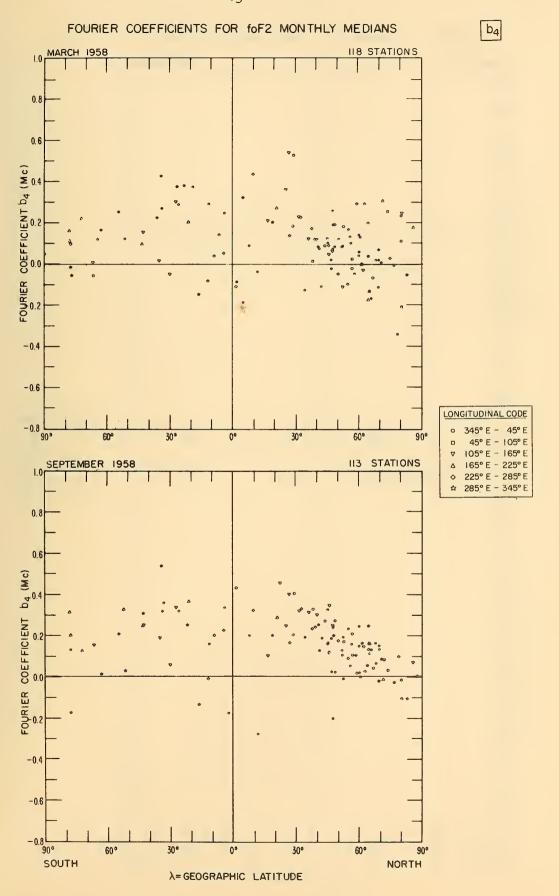


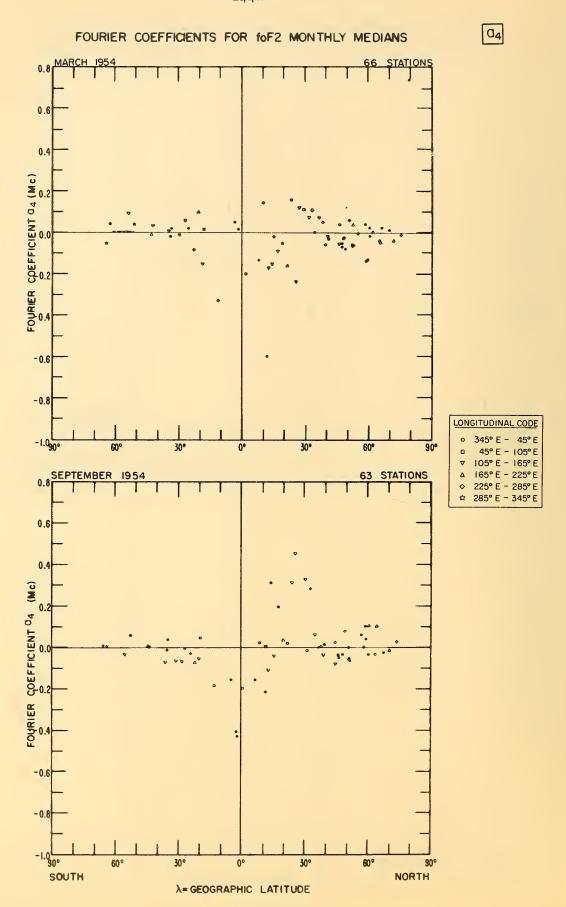


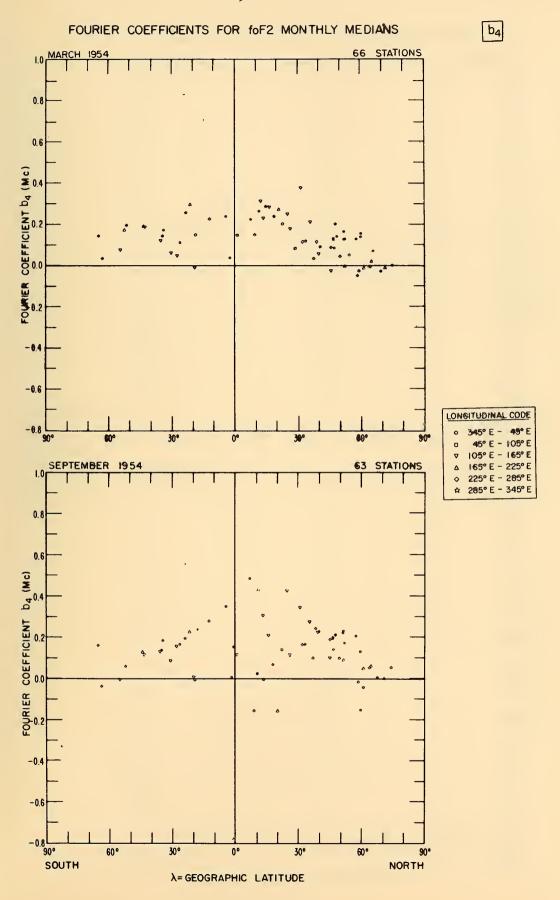


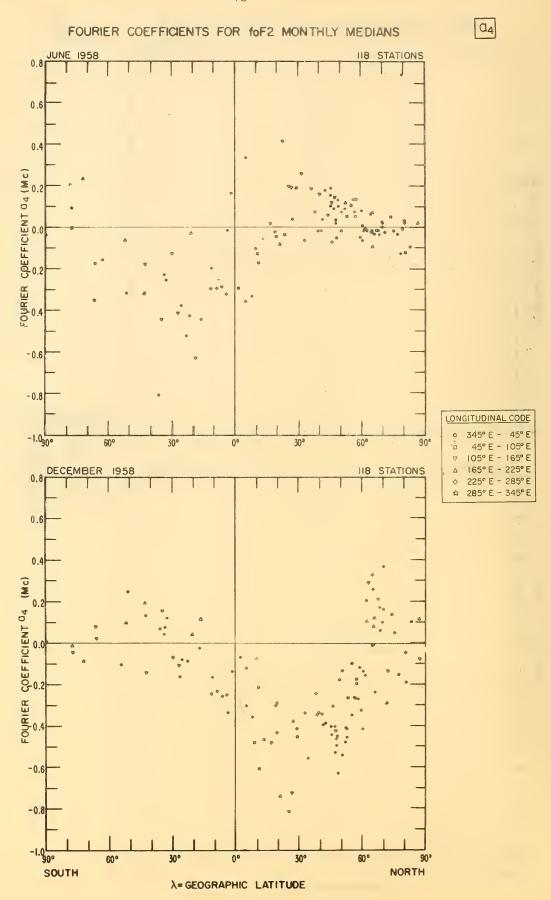


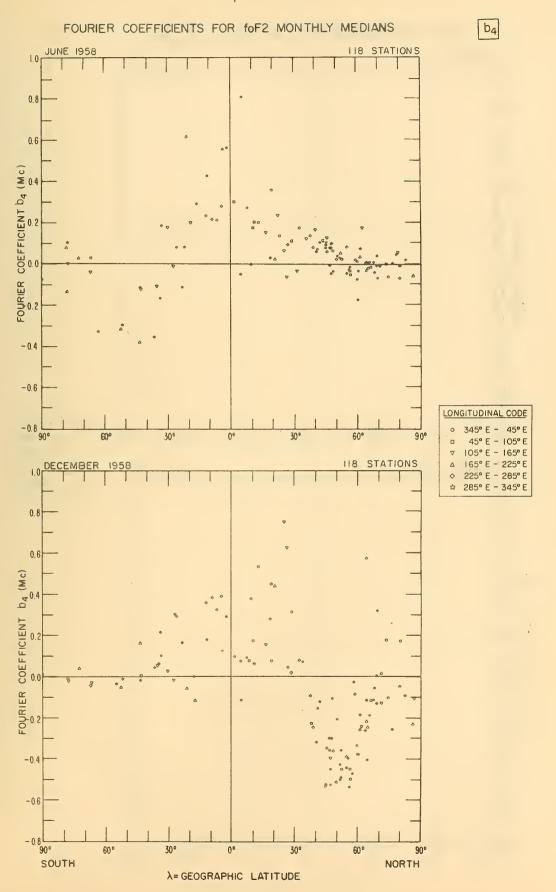


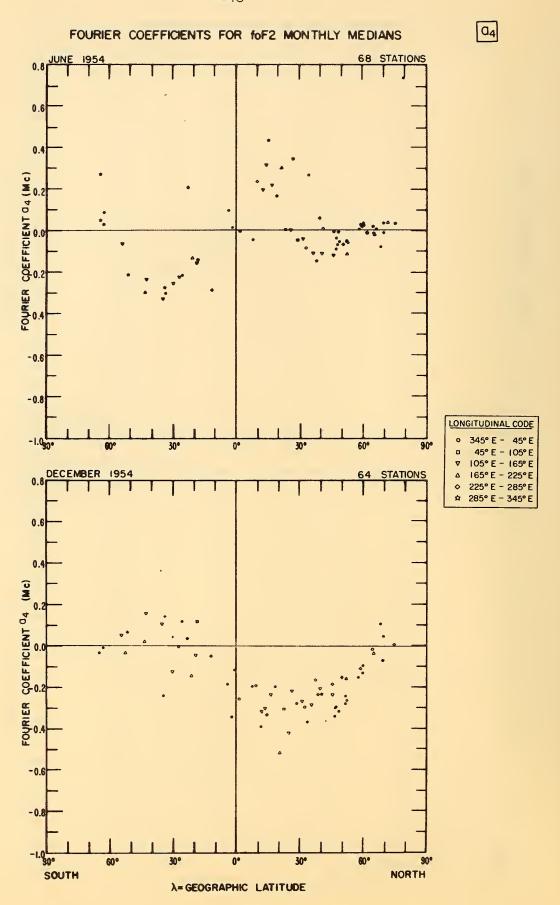


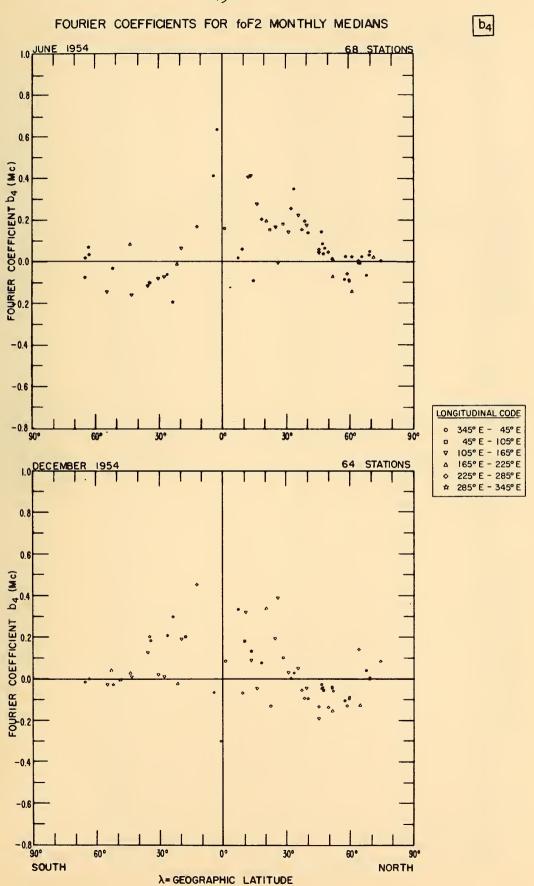


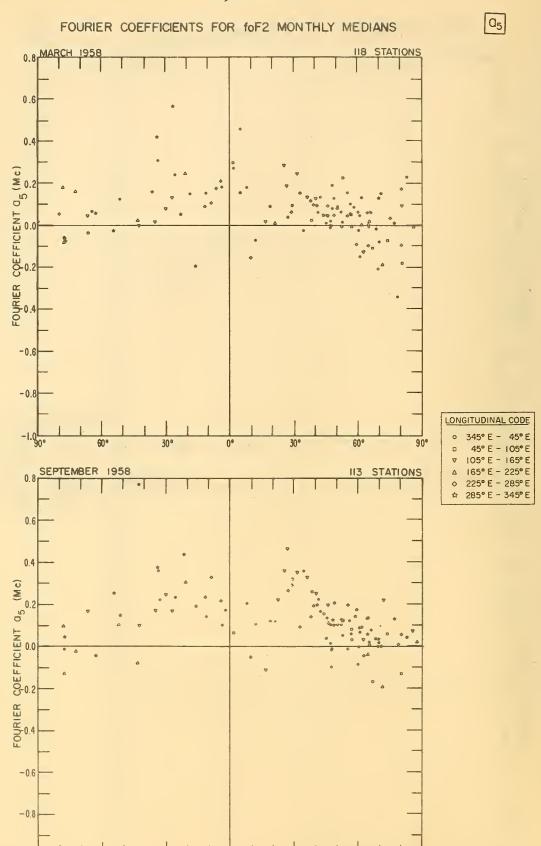












90°

NORTH

60°

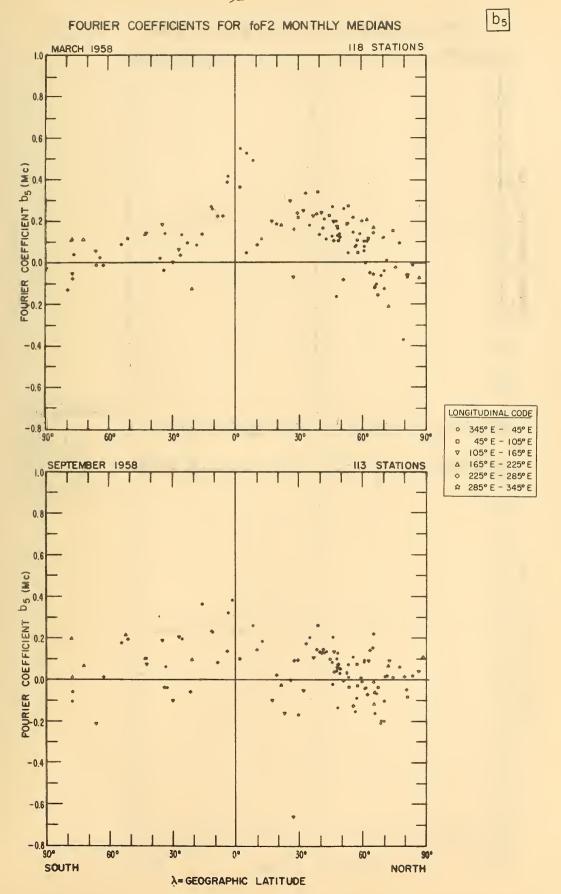
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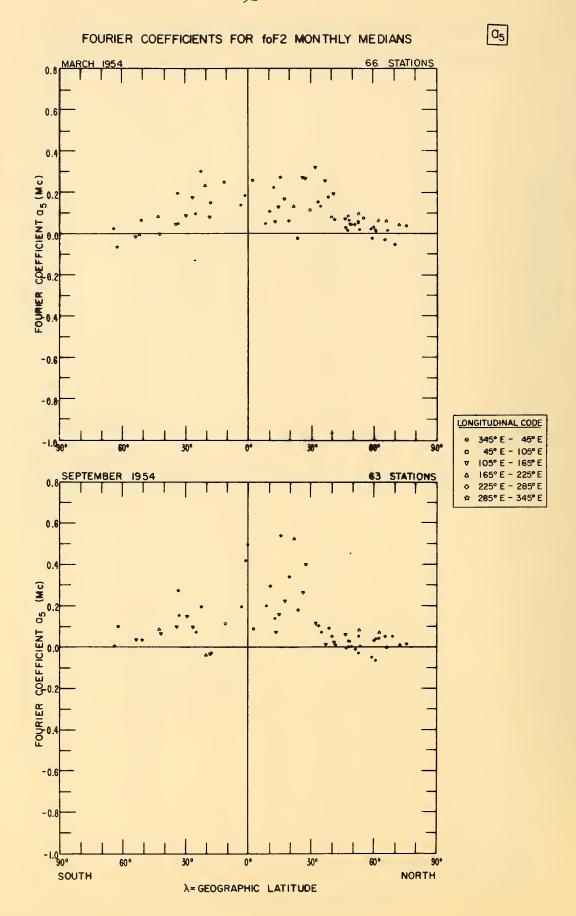
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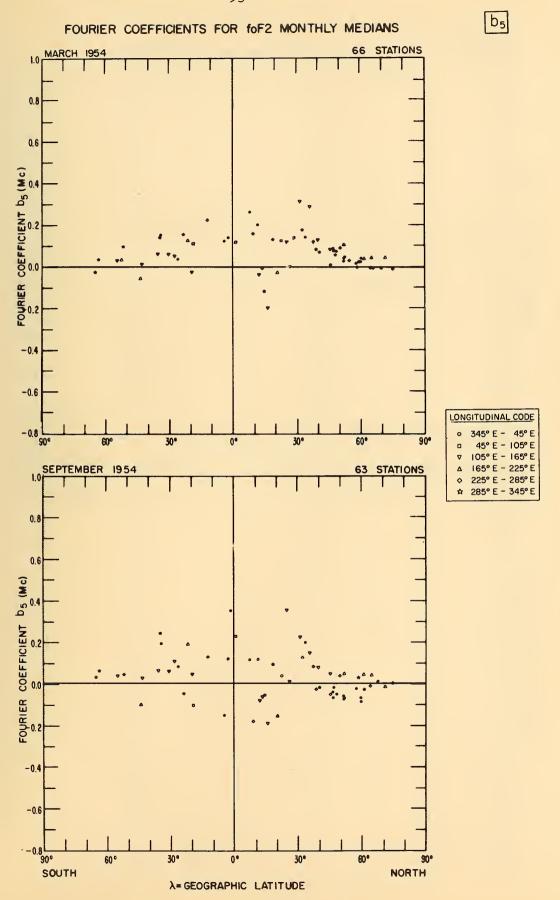
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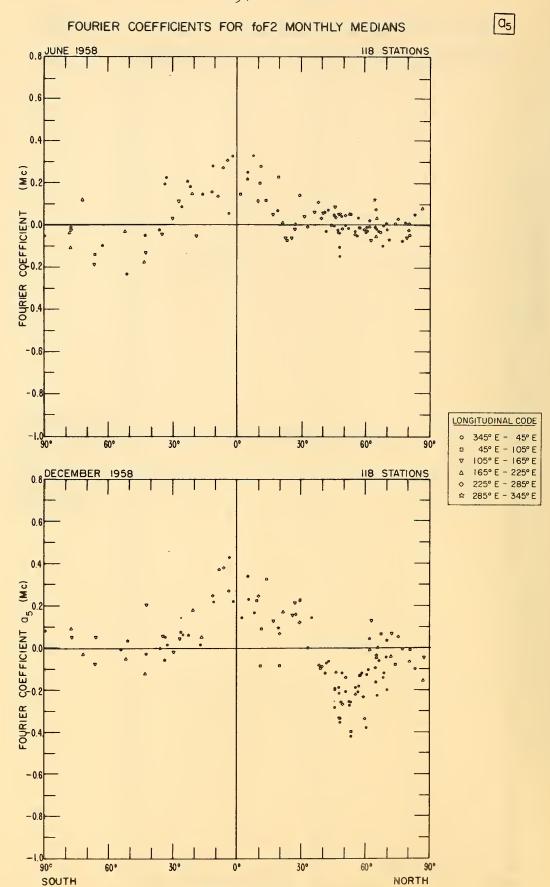
90°

SOUTH

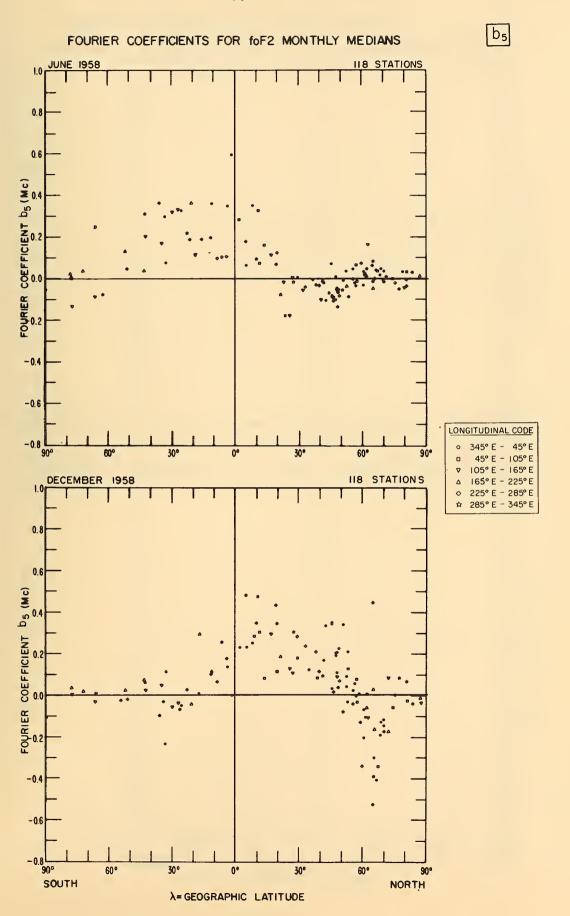


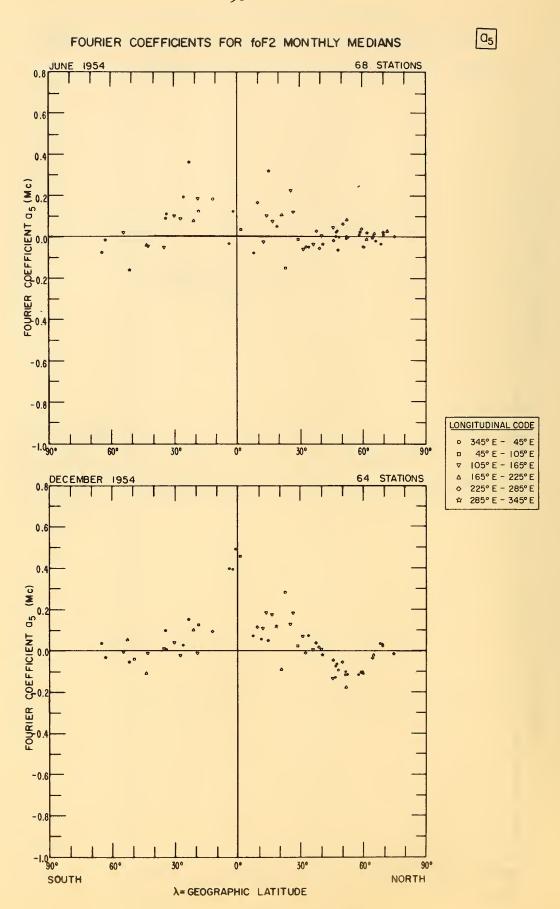


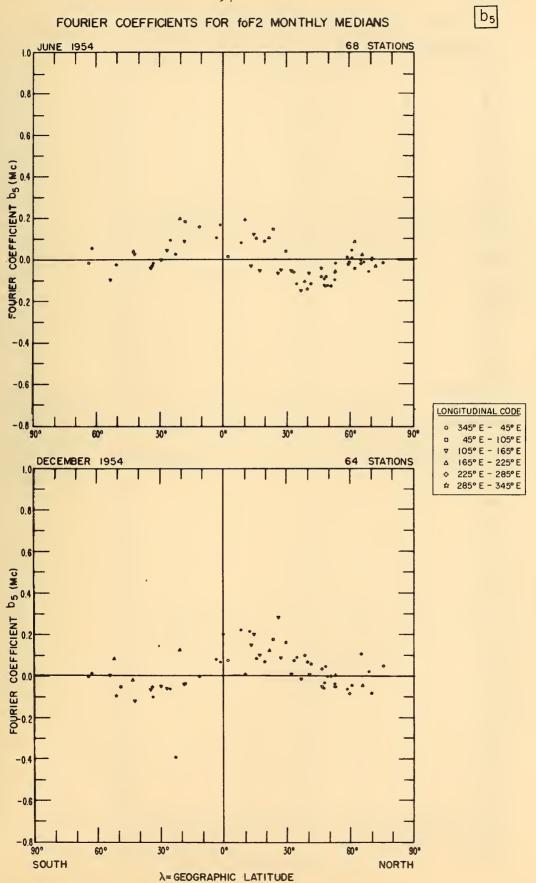


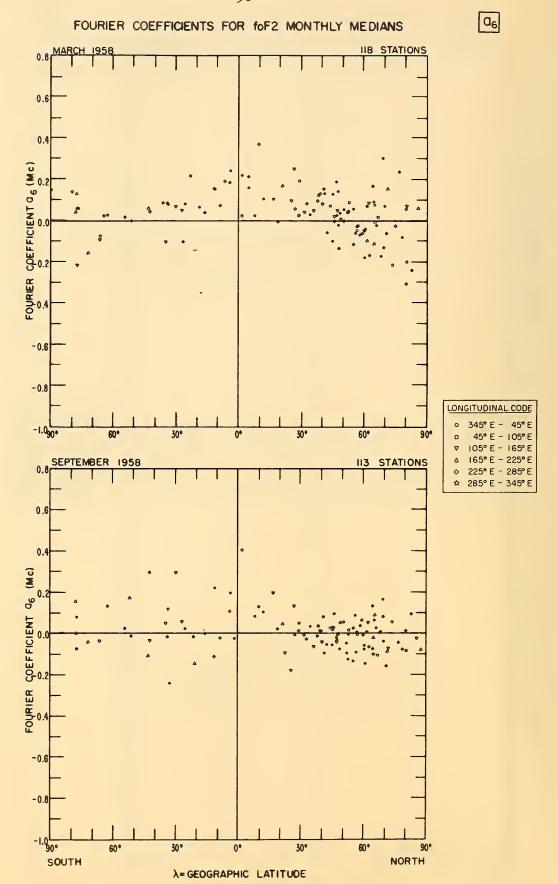


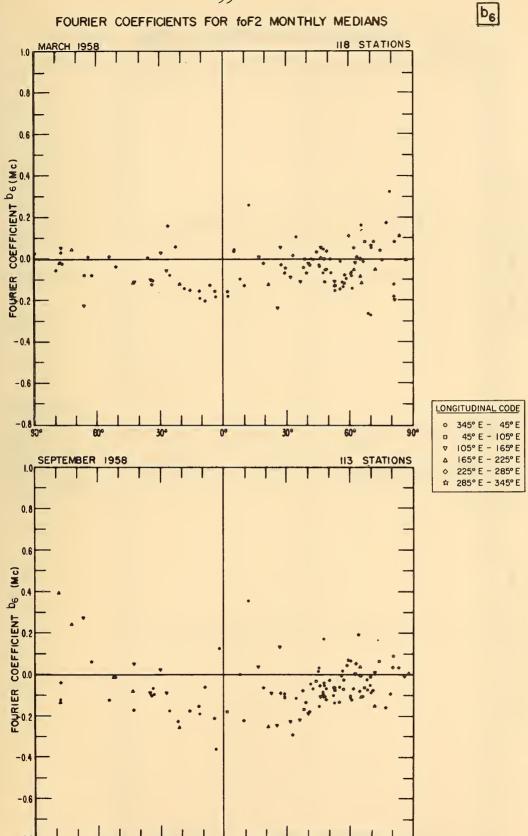
λ= GEOGRAPHIC LATITUDE











30°

60°

901

NORTH

30°

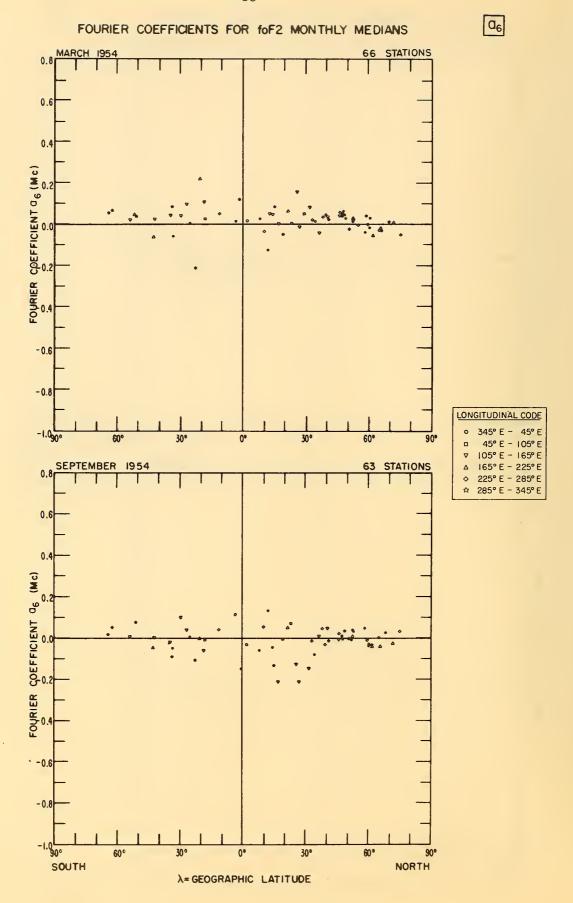
SOUTH

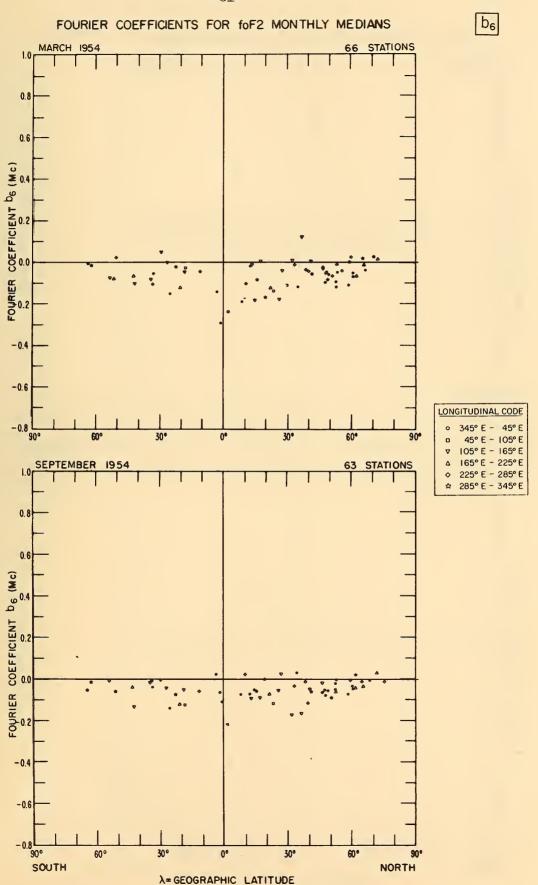
60°

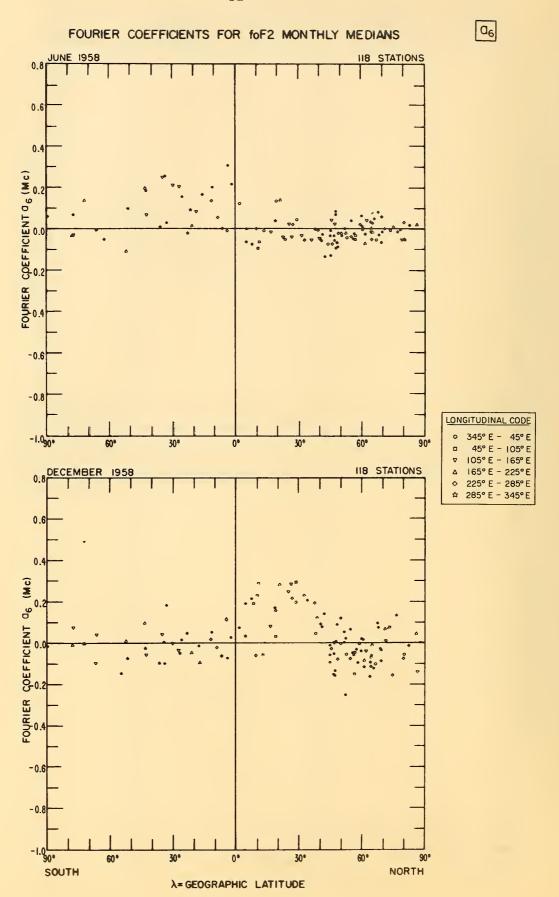
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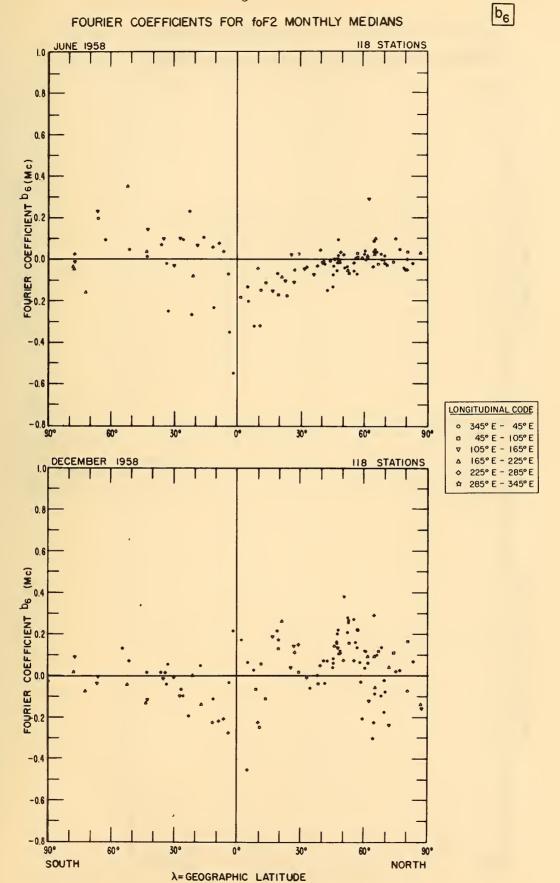
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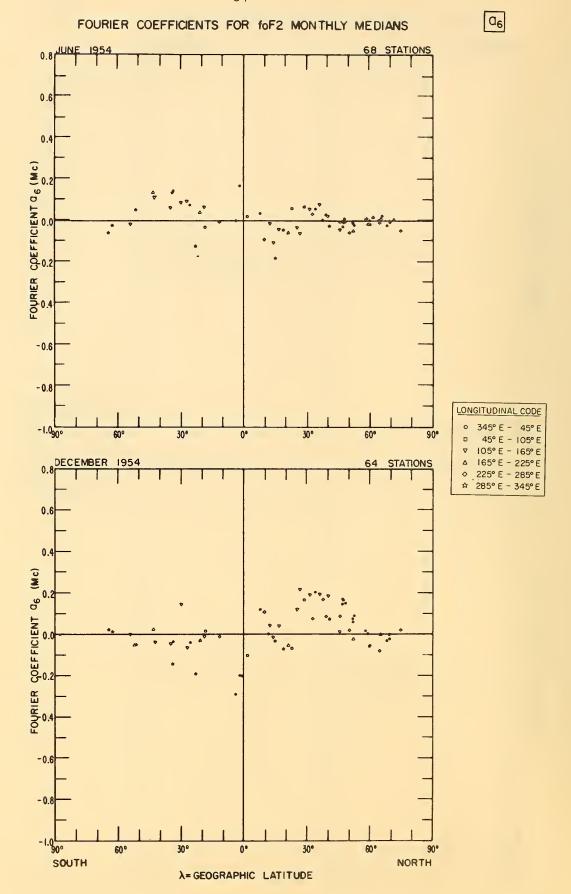
λ= GEOGRAPHIC LATITUDE

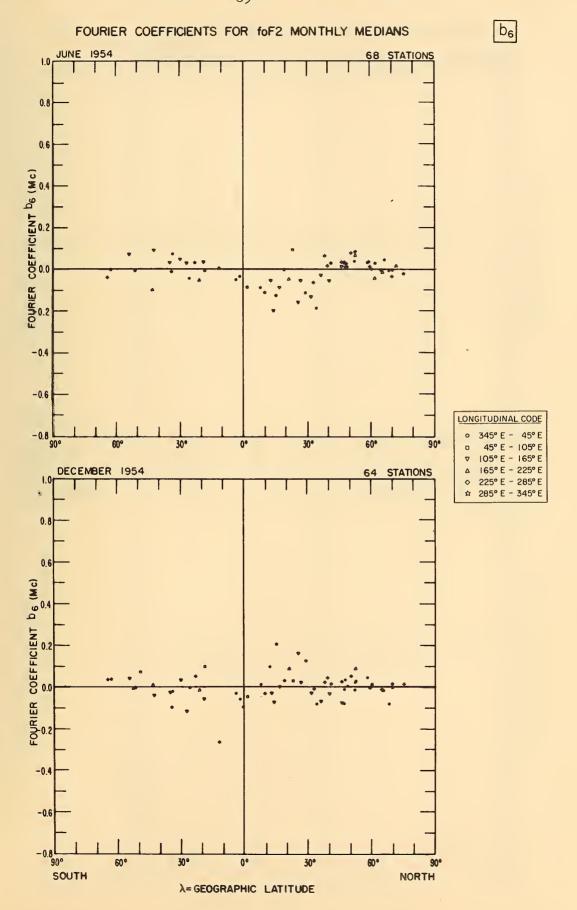


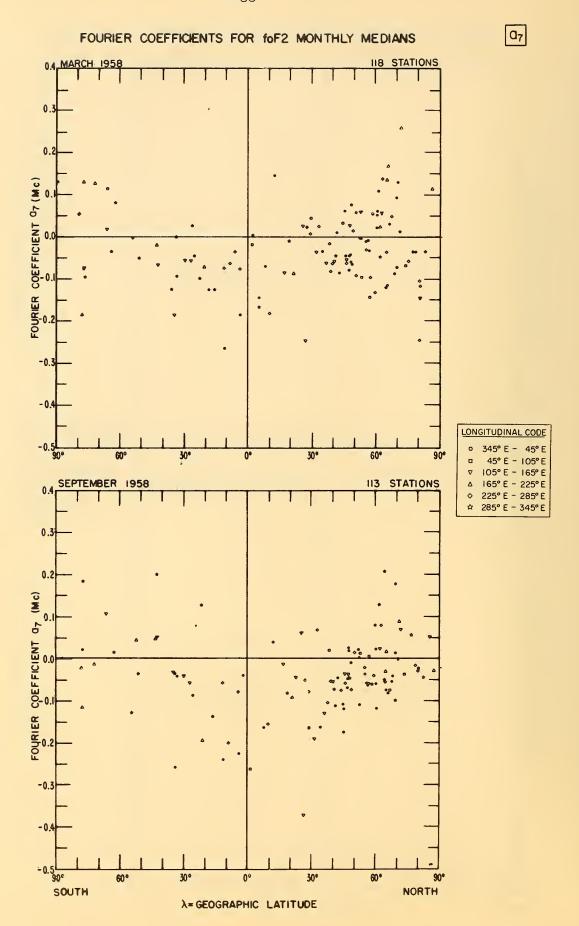


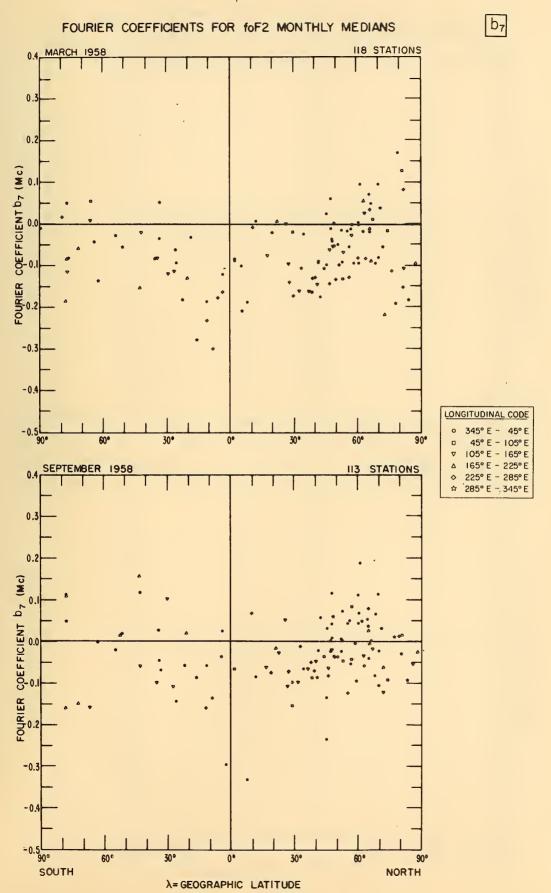


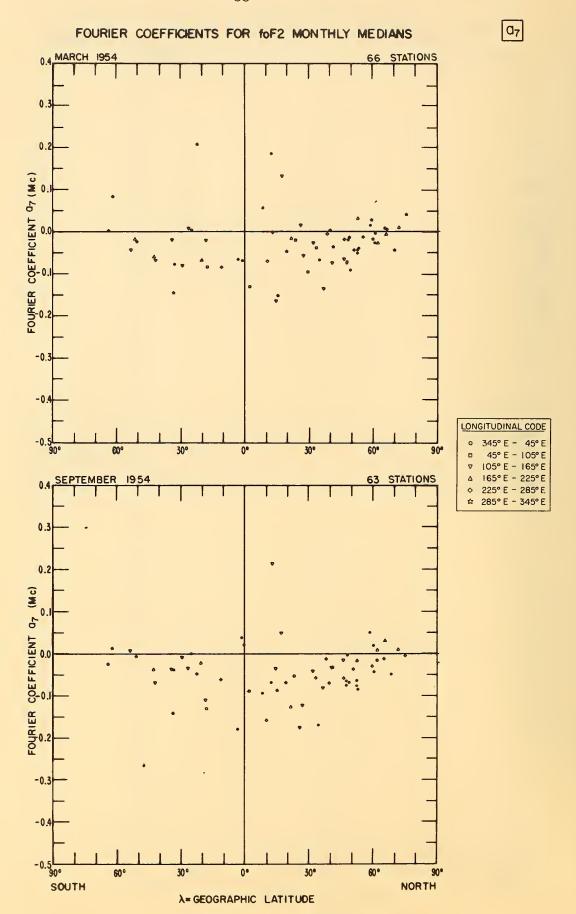


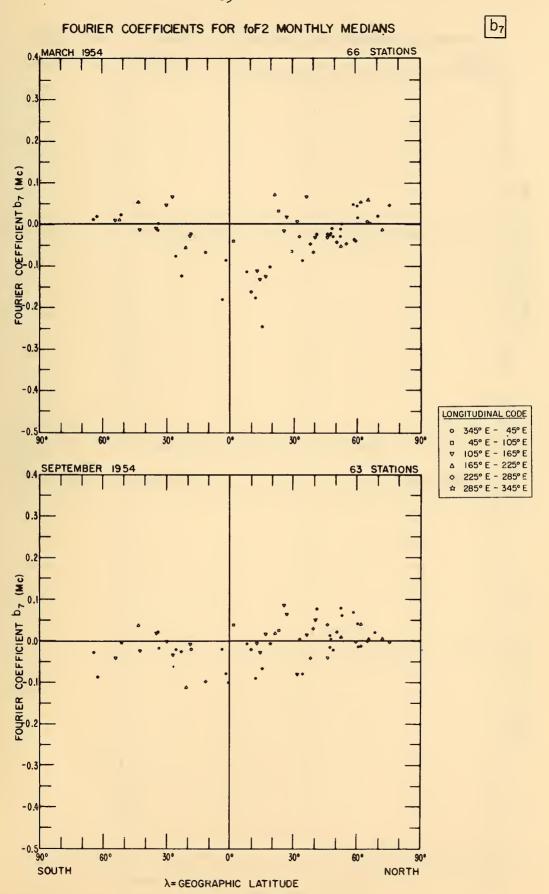


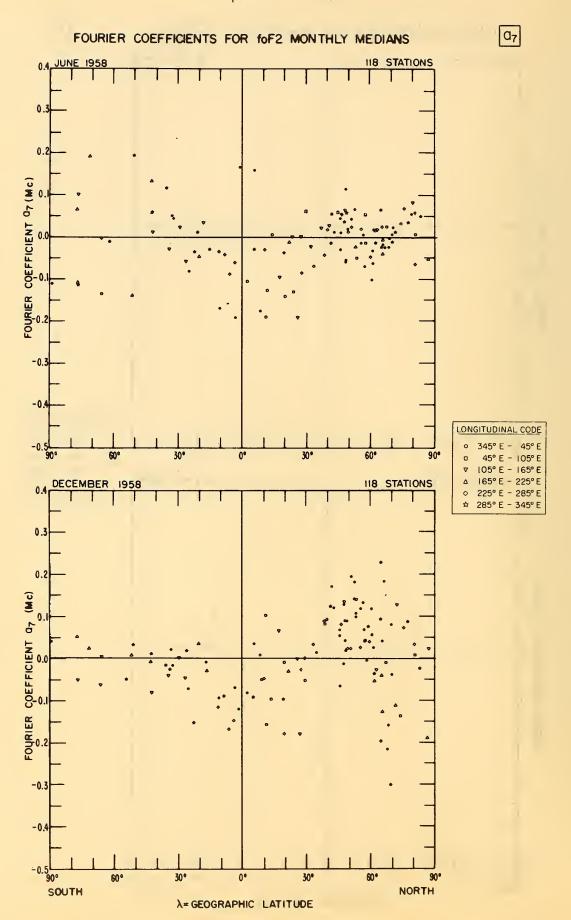


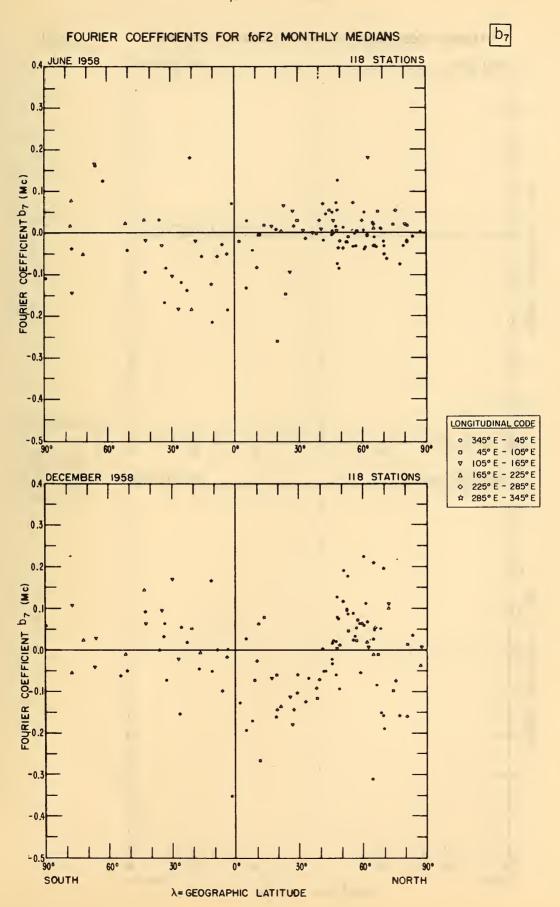


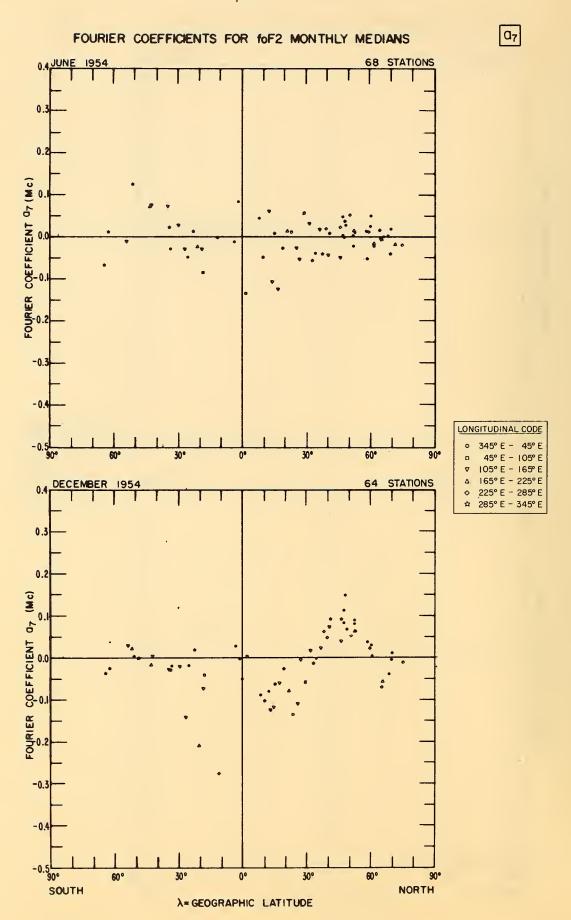


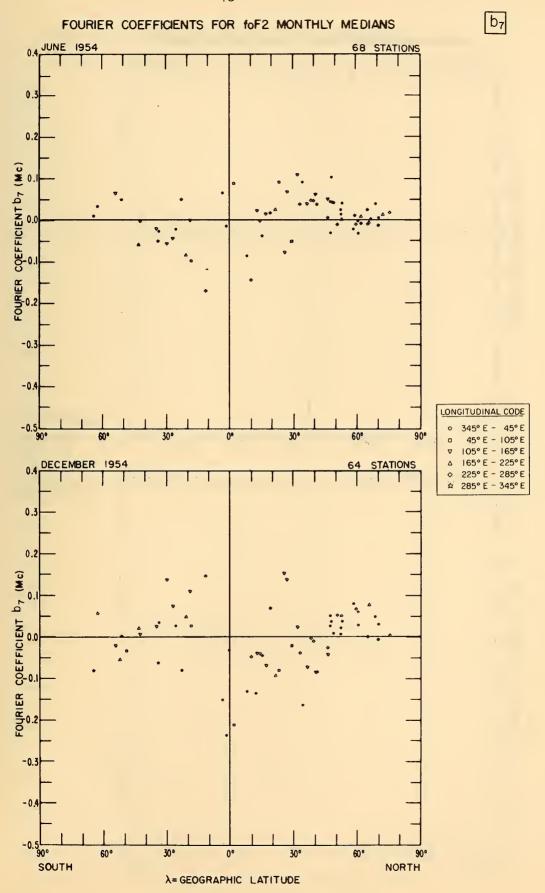


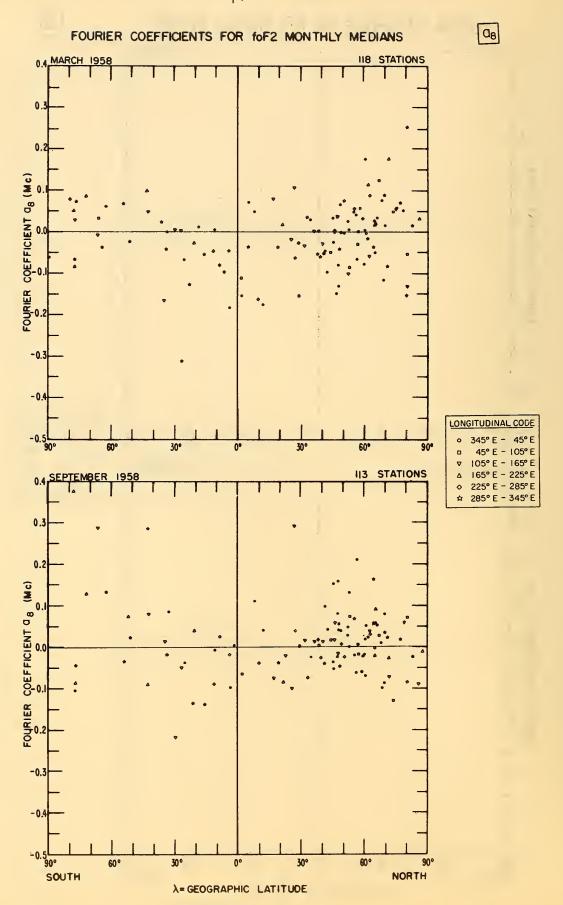


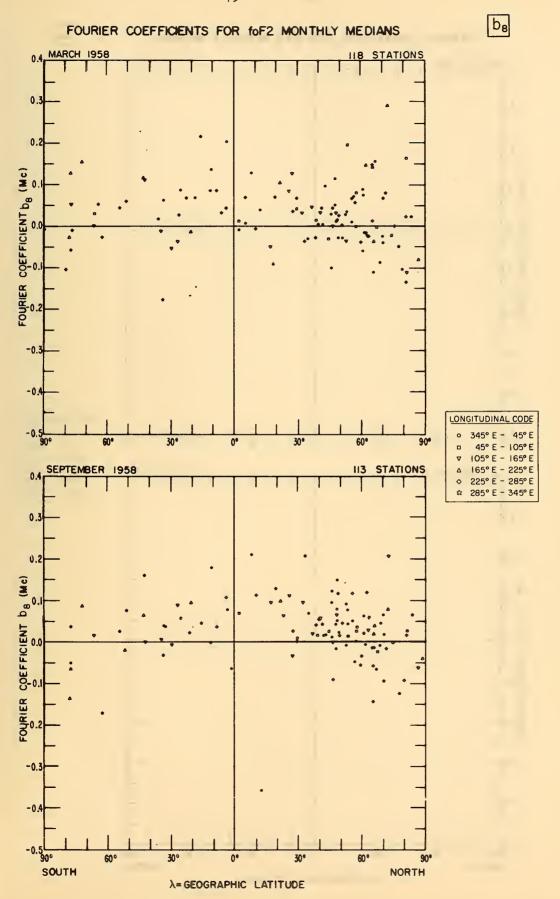


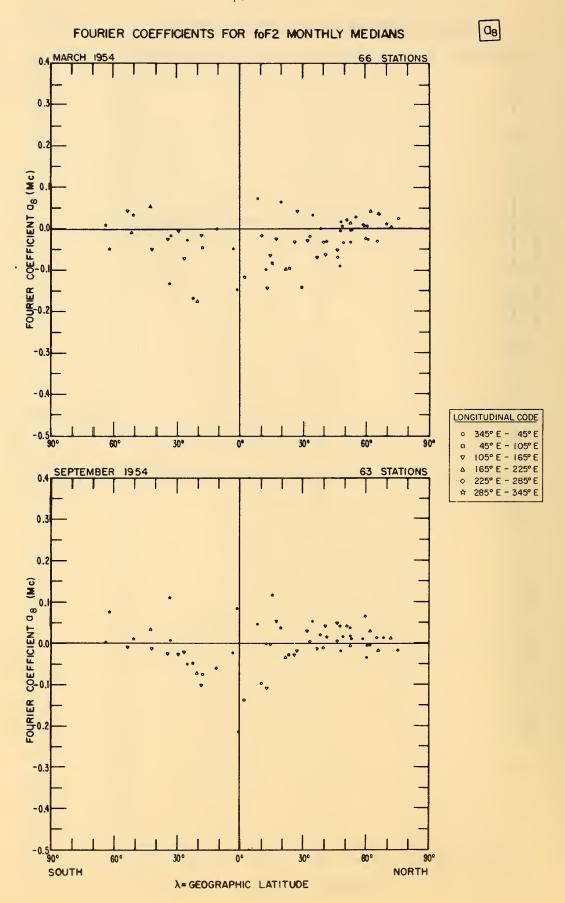


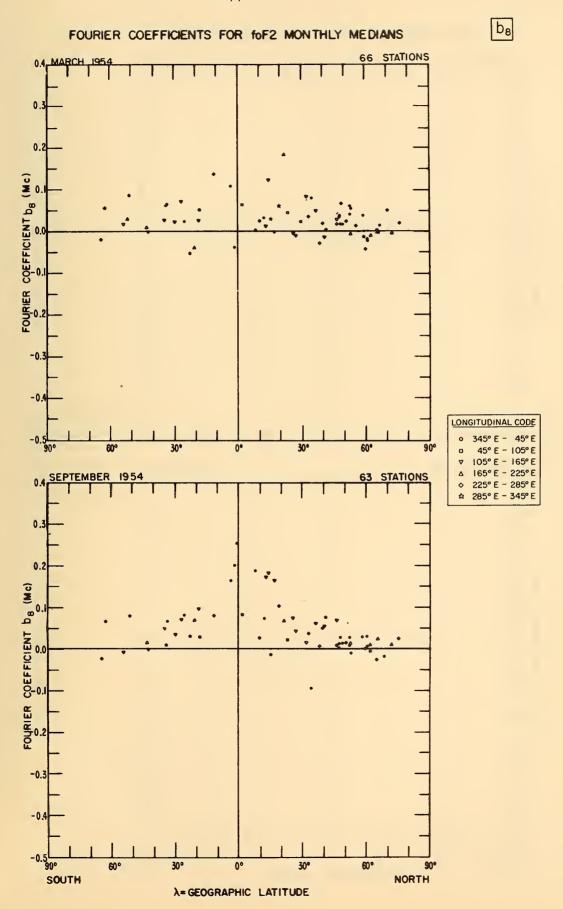


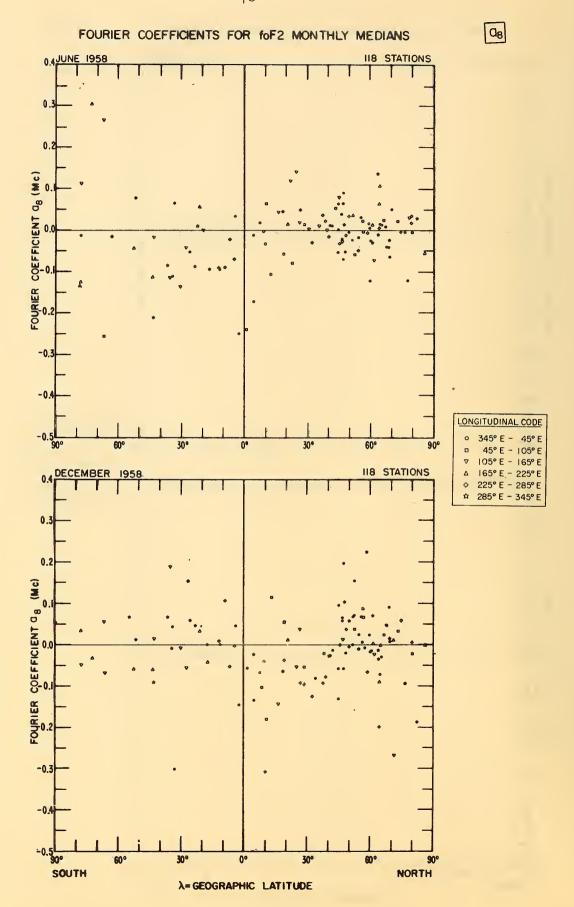


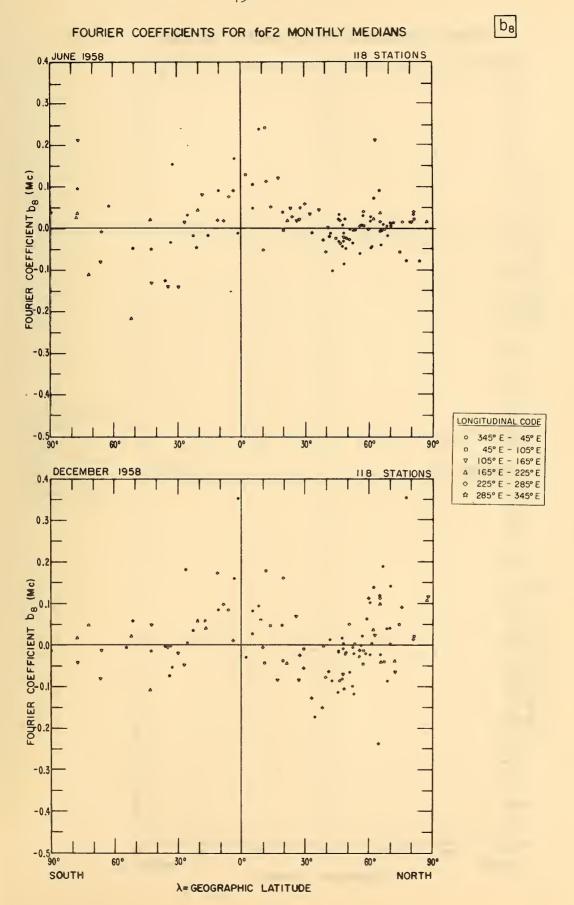


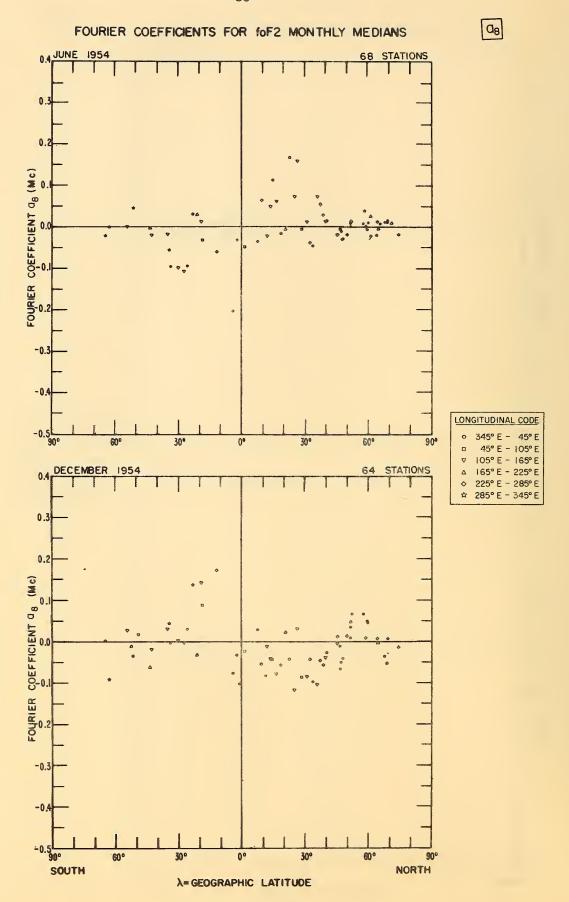


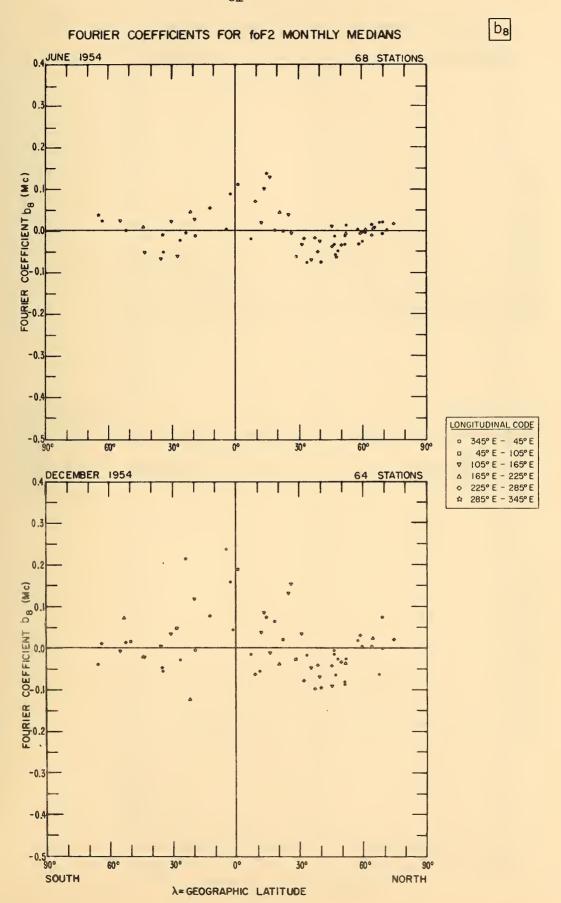


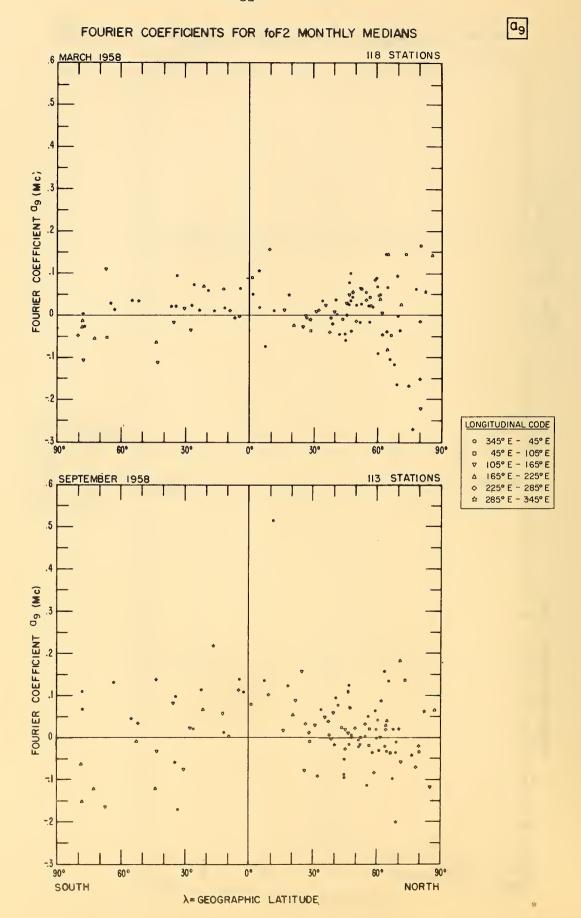


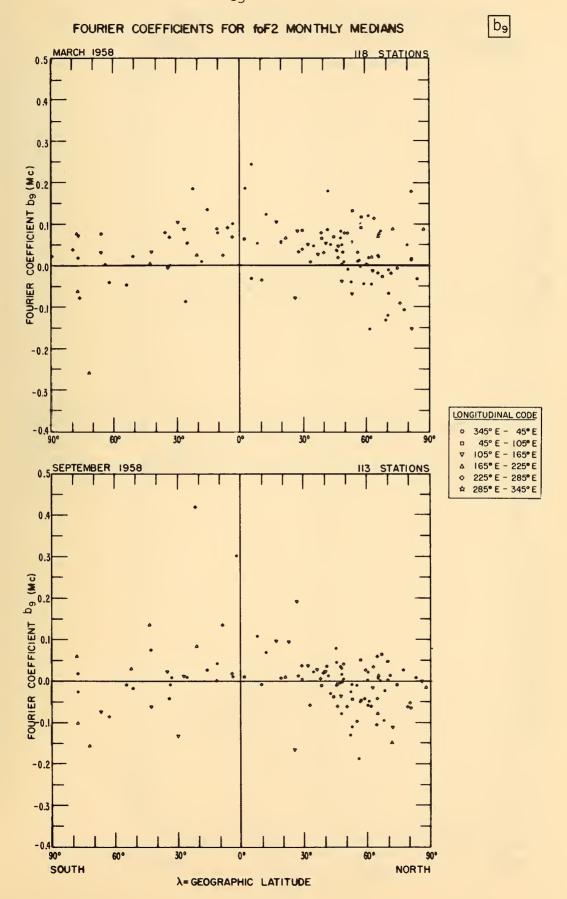


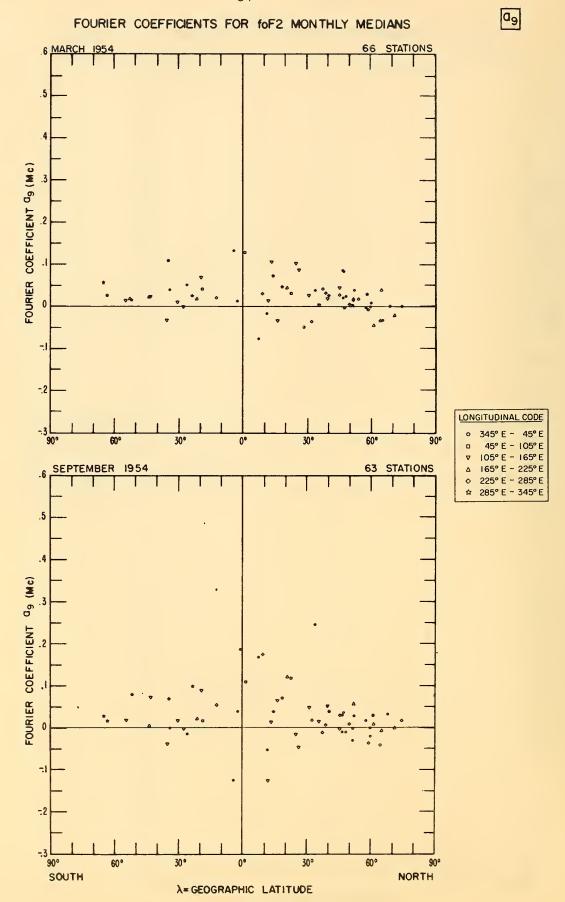


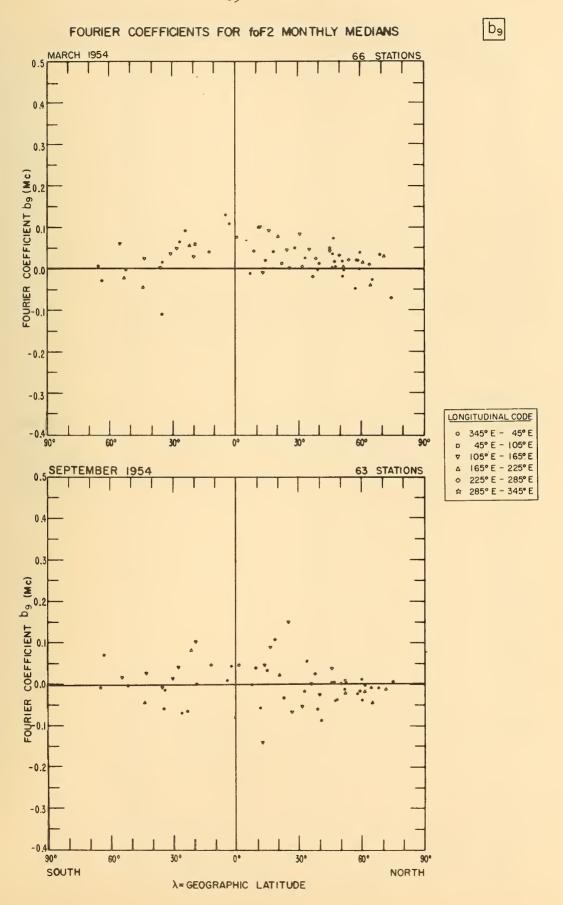


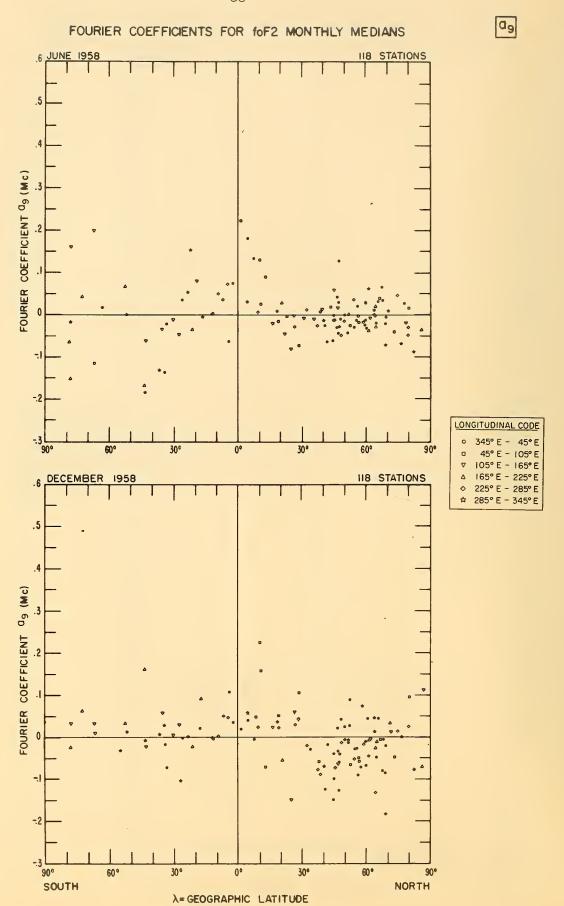


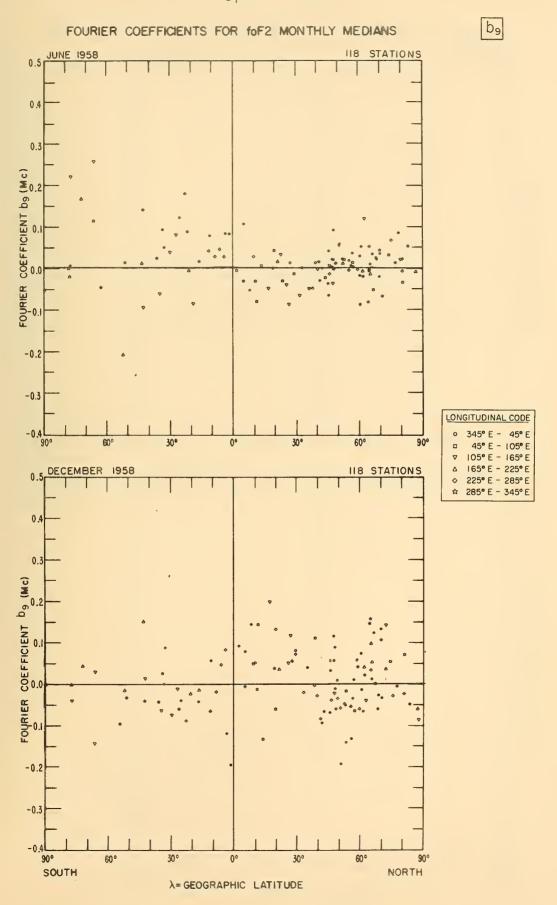


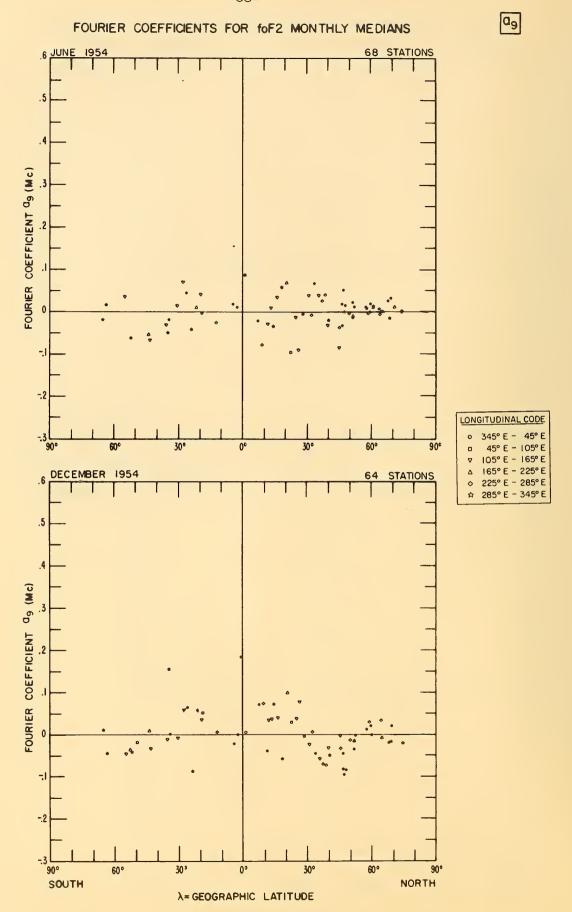


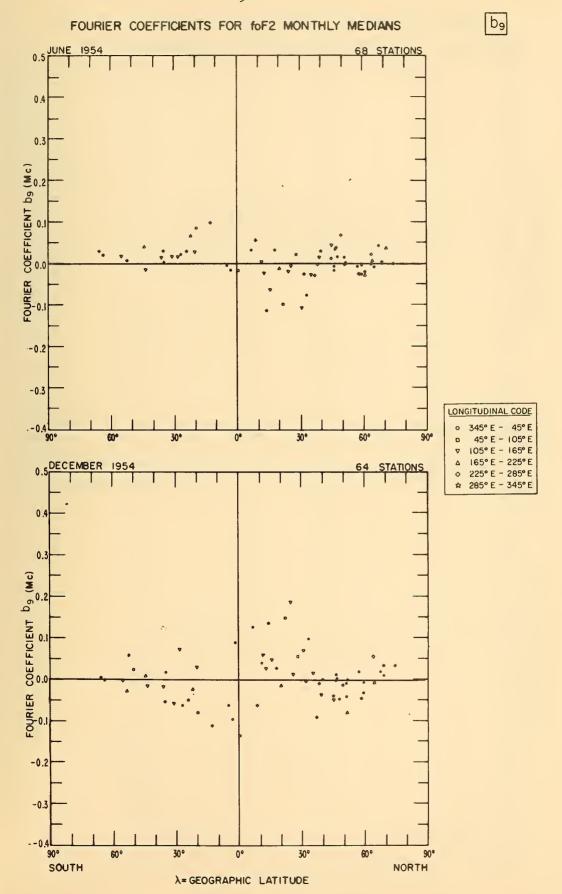


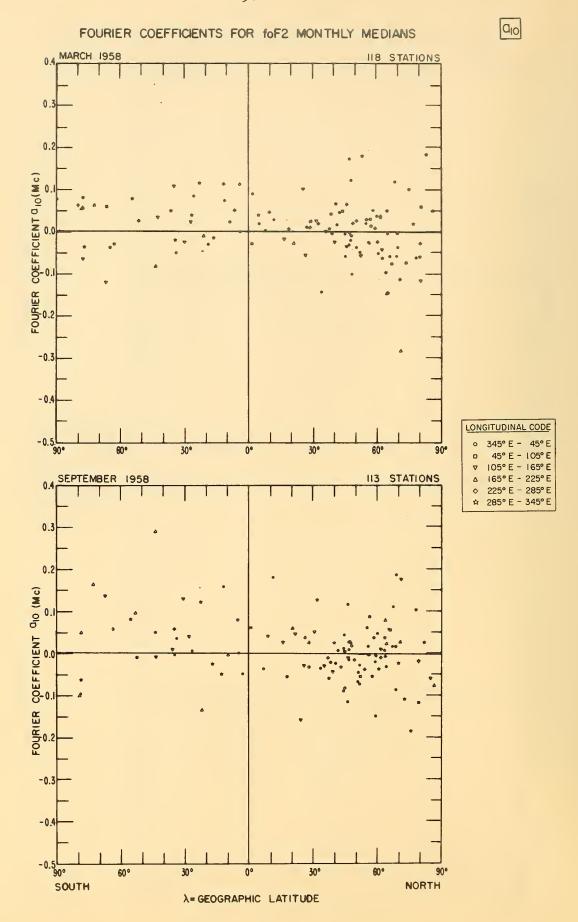


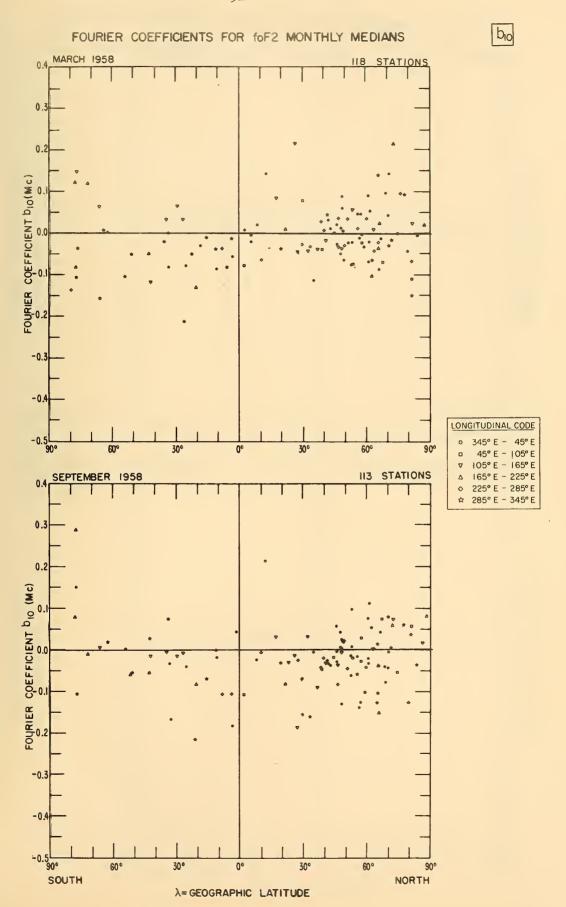


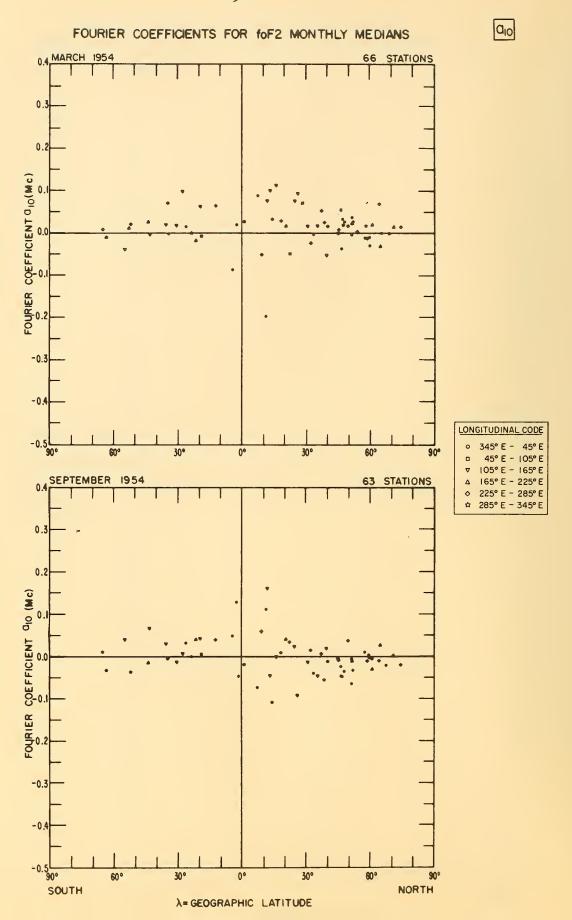


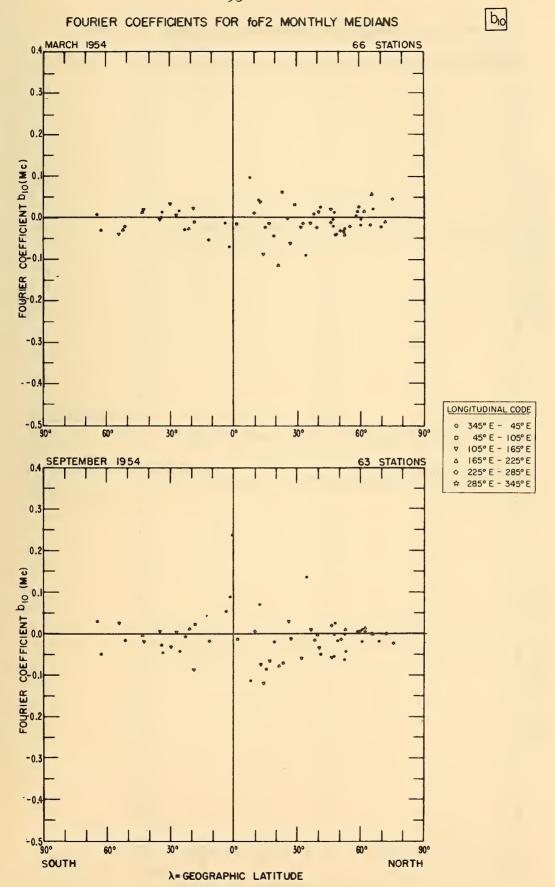


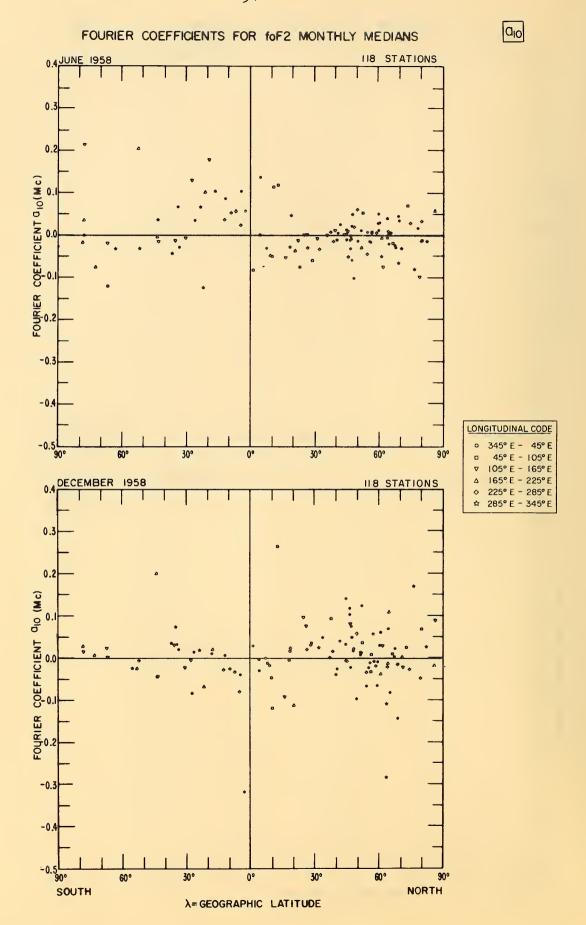


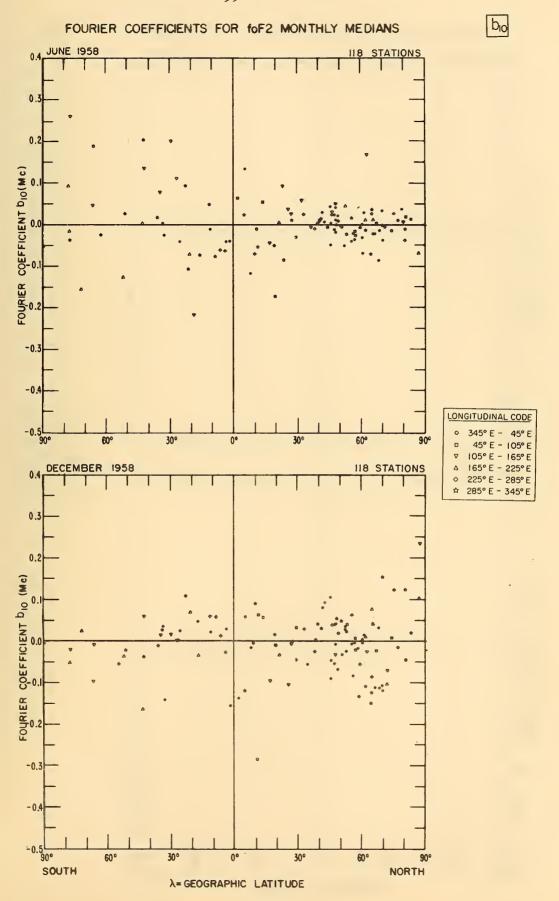


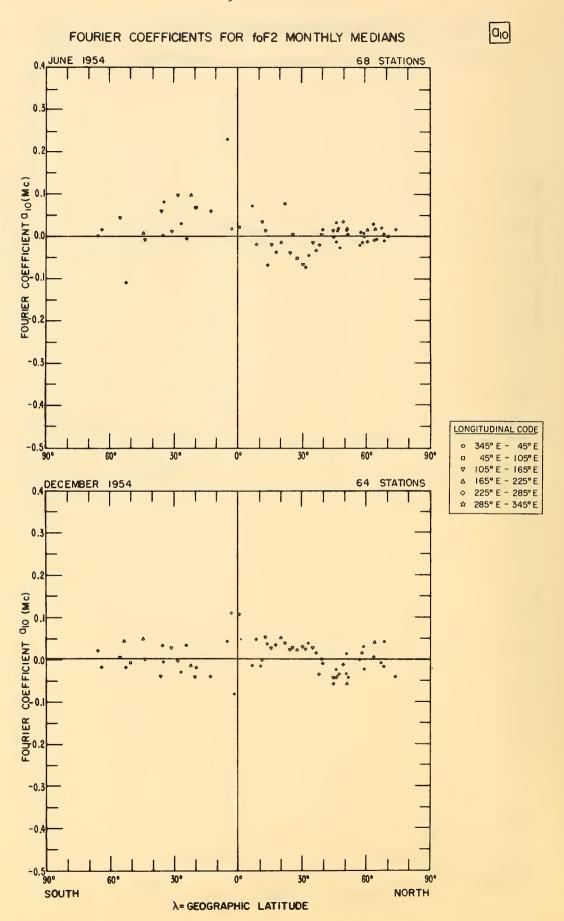


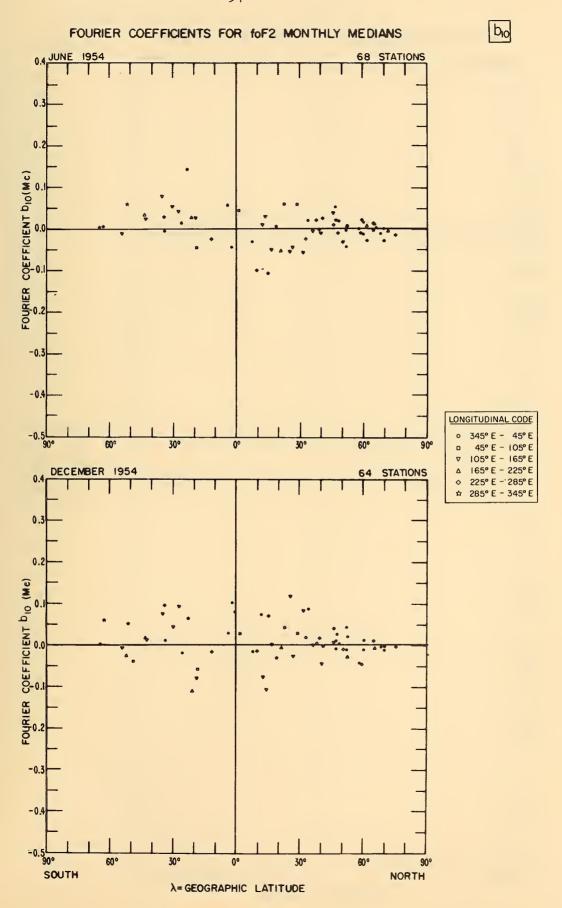


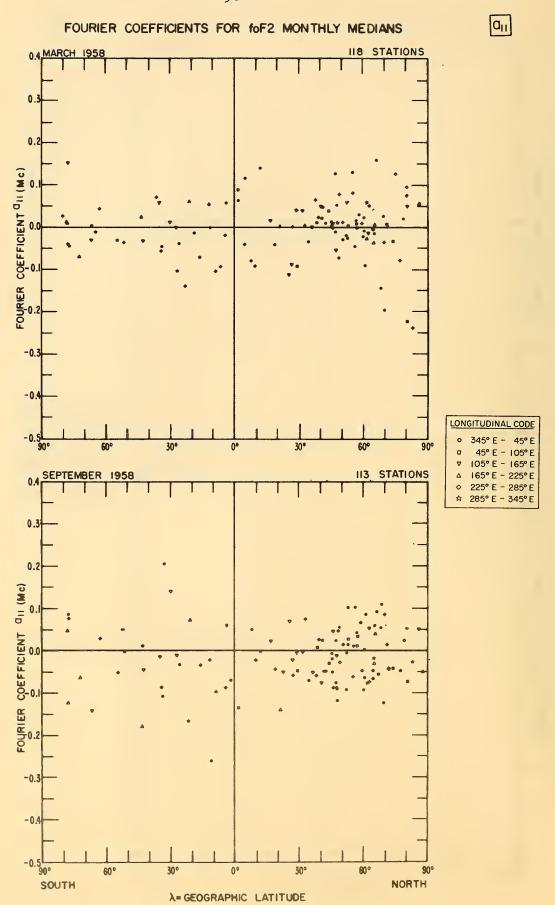


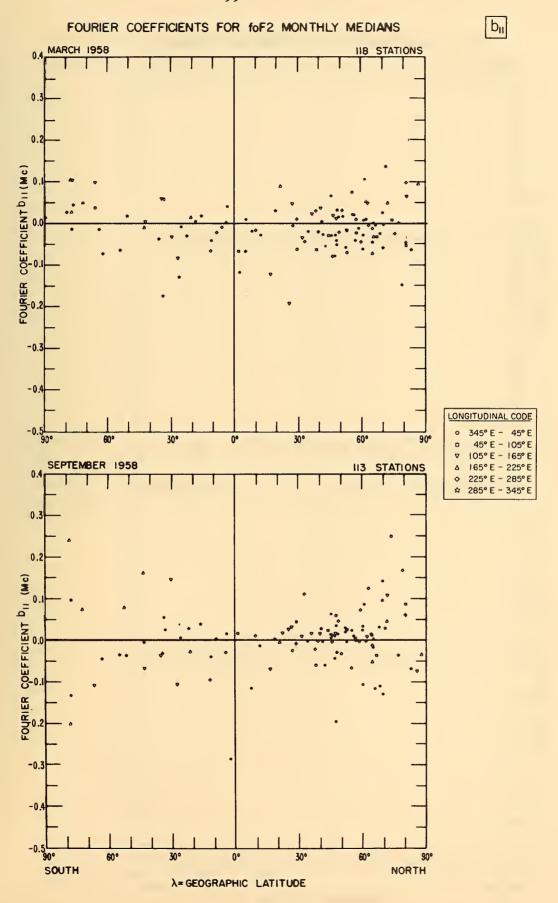


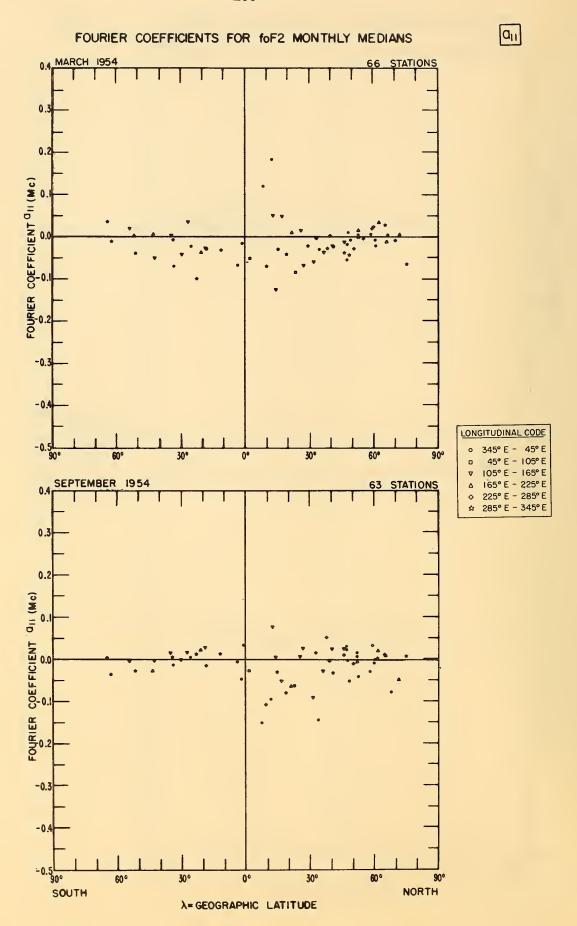


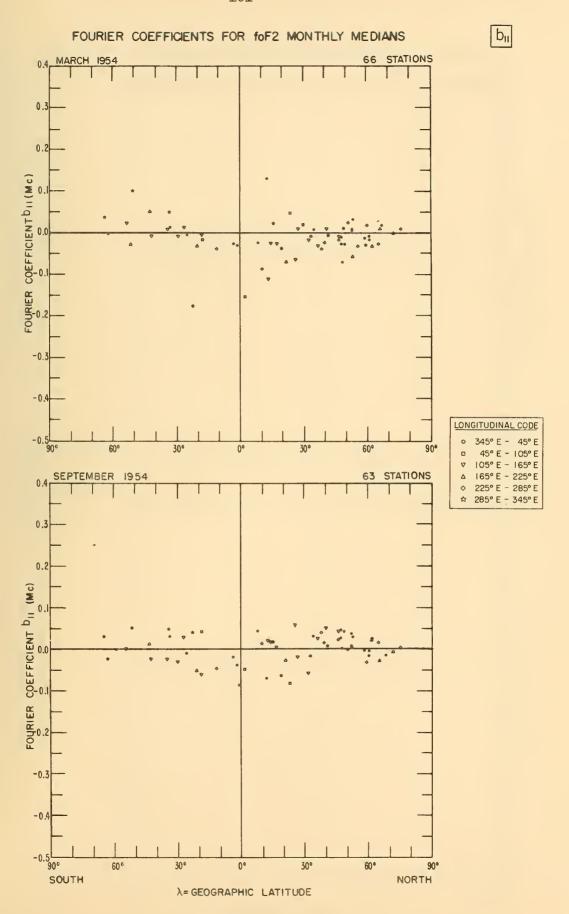


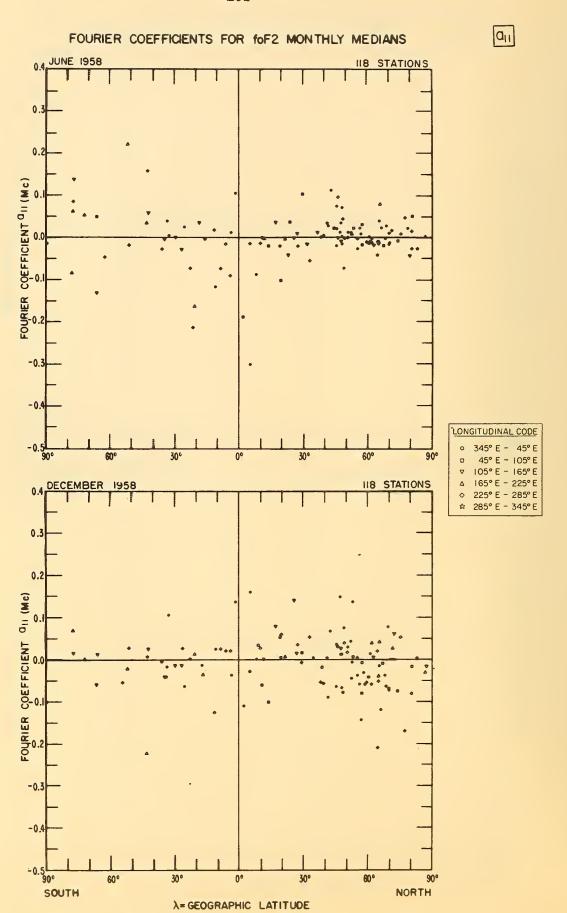


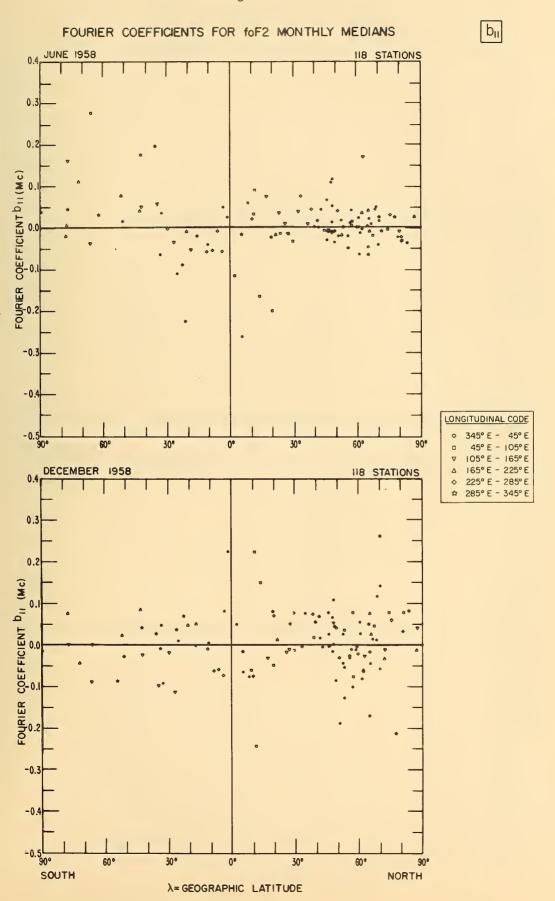


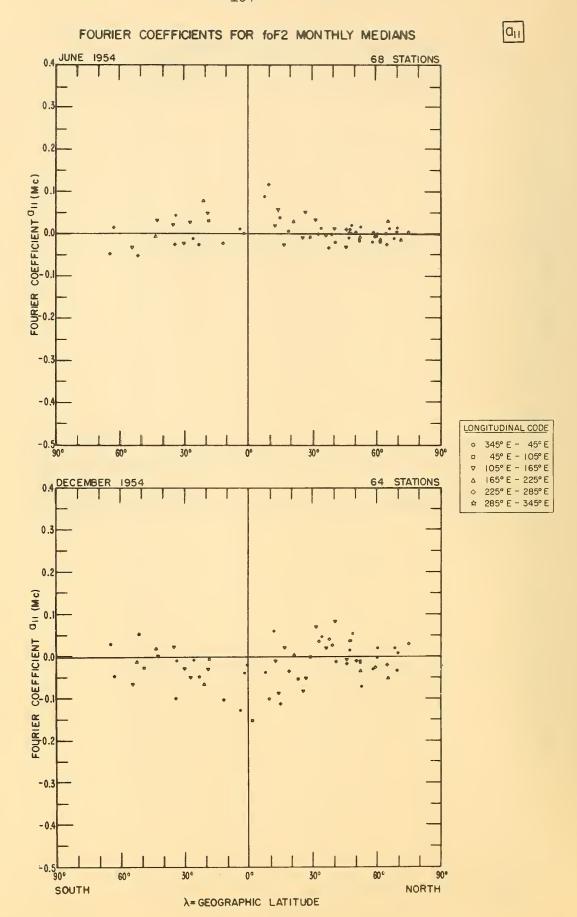


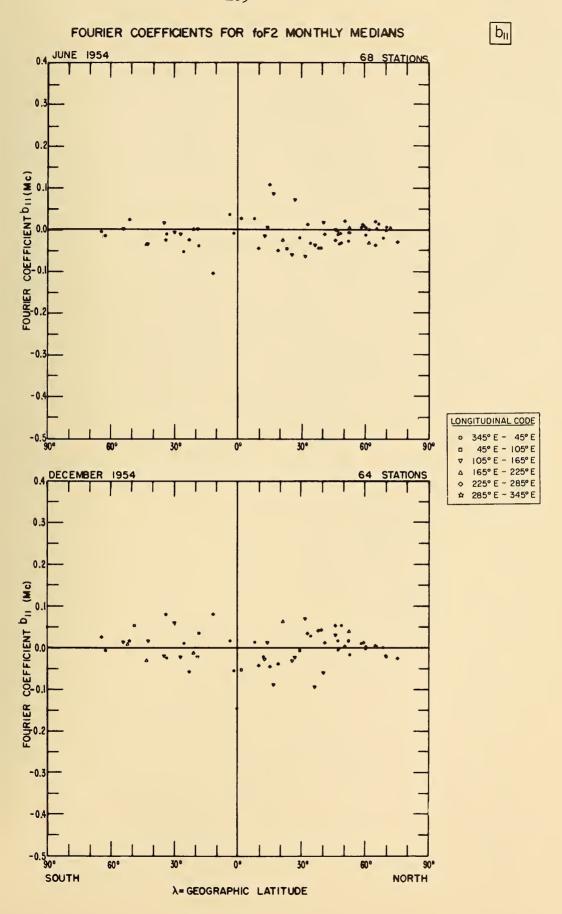














## U. S. DEPARTMENT OF COMMERCE Luther H. Hodges, Secretary

NATIONAL BUREAU OF STANDARDS
A. V. Astin, Director



## THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards at its major laboratories in Washington, D.C., and Boulder, Colorado, is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant publications, appears on the inside of the front cover.

## WASHINGTON, D.C.

Electricity. Resistance and Reactance. Electrochemistry. Electrical Instruments. Magnetic Measurements. Dielectrics. High Voltage.

Metrology. Photometry and Colorimetry. Refractometry. Photographic Research. Length. Engineering Metrology. Mass and Scale. Volumetry and Densimetry.

Ileat. Temperature Physics. Heat Measurements. Cryogenic Physics. Equation of State. Statistical Physics. Radiation Physics. X-ray. Radioactivity. Radiation Theory. High Energy Radiation. Radiological Equipment. Nucleonic Instrumentation. Neutron Physics.

Analytical and Inorganic Chemistry. Pure Substances. Spectrochemistry. Solution Chemistry. Standard Reference Materials. Applied Analytical Research.

Mechanics. Sound. Pressure and Vacuum. Fluid Mechanics. Engineering Mechanics. Rheology. Combustion Controls.

Organic and Fibrous Materials. Rubber. Textiles. Paper. Leather. Testing and Specifications. Polymer Structure. Plastics. Dental Research.

Metallurgy. Thermal Metallurgy. Chemical Metallurgy. Mechanical Metallurgy. Corrosion. Metal Physics. Electrolysis and Metal Deposition.

Mineral Products. Engineering Ceramics. Glass. Refractories. Enameled Metals. Crystal Growth. Physical Properties. Constitution and Microstructure.

Building Research. Structural Engineering. Fire Research. Mechanical Systems. Organic Building Materials. Codes and Safety Standards. Heat Transfer. Inorganic Building Materials.

Applied Mathematics. Numerical Analysis. Computation. Statistical Engineering. Mathematical Physics. Operations Research.

Data Processing Systems. Components and Techniques. Computer Technology. Measurements Automation. Engineering Applications. Systems Analysis.

Atomic Physics. Spectroscopy. Infrared Spectroscopy. Solid State Physics. Electron Physics. Atomic Physics. Instrumentation. Engineering Electronics. Electron Devices. Electronic Instrumentation. Mechanical Instruments. Basic Instrumentation.

Physical Chemistry. Thermochemistry. Surface Chemistry. Organic Chemistry. Molecular Spectroscopy. Molecular Kinetics. Mass Spectrometry.

Office of Weights and Measures.

## BOULDER, COLO.

Cryogenic Engineering. Cryogenic Equipment. Cryogenic Processes. Properties of Materials. Cryogenic Technical Services.

Ionosphere Research and Propagation. Low Frequency and Very Low Frequency Research. Ionosphere Research. Prediction Services. Sun-Earth Relationships. Field Engineering. Radio Warning Services. Vertical Soundings Research.

Radio Propagation Engineering. Data Reduction Instrumentation. Radio Noise. Tropospheric Measurements. Tropospheric Analysis. Propagation-Terrain Effects. Radio-Meteorology. Lower Atmosphere Physics.

Radio Standards. High Frequency Electrical Standards. Radio Broadcast Service. Radio and Microwave Materials. Atomic Frequency and Time Interval Standards. Electronic Calibration Center. Millimeter-Wave Research. Microwave Circuit Standards.

Radio Systems. Applied Electromagnetic Theory. High Frequency and Very High Frequency Research. Modulation Research. Antenna Research. Navigation Systems.

Upper Atmosphere and Space Physics. Upper Atmosphere and Plasma Physics. Ionosphere and Exosphere Scatter. Airglow and Aurora. lonospheric Radio Astronomy.

PART PROMICE

